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BY

EDWARD L. NICHOLS,
ERNEST MERRITT, and FREDERICK BEDELL

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By EDWARD L. NICHOLS

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THE

PHYSICAL REVIEW.

REACTION EFFECTS PRODUCED BY THE DISCHARGE OF ELECTRICITY FROM POINTS IN GASES AND THE BEARING OF THESE EFFECTS ON THE THEORY OF THE SMALL ION.[1]

By Edward J. Moore.

ARRHENIUS[2] published in 1897 a series of observations on the reaction effects produced when electricity discharges from points in gases. He found that the ratio of the reaction to the current was very nearly constant and that the force of reaction was approximately inversely proportional to the pressure when the current was constant.

His results are discussed by J. J. Thomson[3] from the standpoint of the electron theory and the following equation developed for the force of reaction on the point:

$$F = \frac{i}{K}\int dz, \qquad (1)$$

where i is the current, z the direction of the normal from the point to the plane and K the velocity of the ion in unit field. This equation assumes that ions of one sign only are present in the region between point and plane, that these ions are formed in the immediate neighborhood of the point, and travel without change in character to the plane opposite. It seems improbable, as Thomson himself points out, that both these conditions are strictly fulfilled in practice. This investigation was undertaken in order to see to what extent the above relations would hold under the experimental condition that must be imposed. It was hoped that even if the method did not prove to be capable of yielding satis-

[1] A preliminary report of this paper was presented at the Minneapolis meeting of the Physical Society, December, 1910.

[2] Wied. Ann., Vol. 63, p. 305, 1897.

[3] Conduction of Electricity through Gases, 2d ed., p. 507.

factory results for the absolute values of the mobilities it would, nevertheless, yield consistent relative values and in that way, serve to throw light on the interesting and much discussed question of the character of the gaseous ion.

THE APPARATUS AND THE METHOD.

Two forms of the apparatus were used. The arrangement was very simple. In the first form two fine sewing needles were fastened to a wire and formed into a couple (see Fig. 1). This couple was suspended by a phosphor bronze ribbon in such a way that electricity could be made to discharge from the points to the planes P and P' placed opposite. These planes were five cm. in diameter and were connected to earth through a

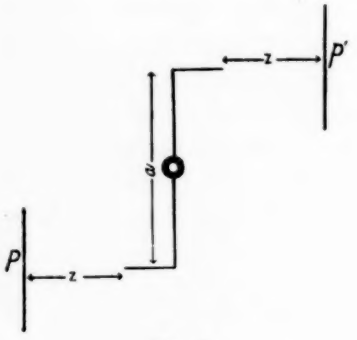

Fig. 1.

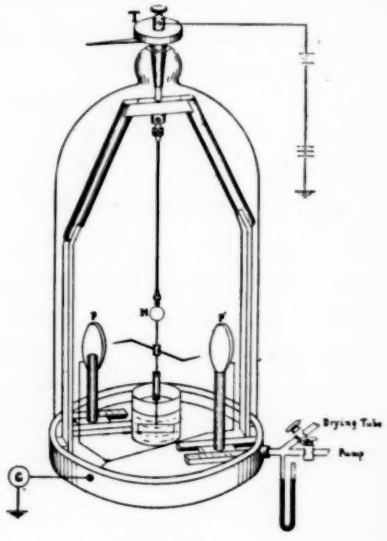

Fig. 2.

galvanometer G, Fig. 2. The distance z between point and plane could be varied between 0 and 5.5 cm. The position of the couple could be read by means of the mirror M. The force of reaction for a given value of z could be measured by the torsion of the suspension, and at the same time the current flowing from the points could be measured by the galvanometer. The apparatus was placed under a large bell jar and arranged so as to admit of varying the pressure. The inside of the bell jar was covered with tin-foil and connected to earth. Electrical connection with the points was effected through a stop cock T in the top of the bell jar. This stop cock served also as a torsion head for measuring the reaction.

The second form differed from the first in having, as it were, two

couples (see Fig. 3) instead of one. The planes were replaced by a cylindrical brass ring 18 cm. in diameter and 5 cm. wide. The ring was connected to earth through the galvanometer and observations were made in precisely the same way as with the single couple. This form of the apparatus was well adapted for measuring the mobility with varying pressures, and yielded curves identical in form with those obtained with the single couple. An aluminum vane, connected to the couple by a piece of ebonite and dipped into a vessel of oil, served to damp the couple.

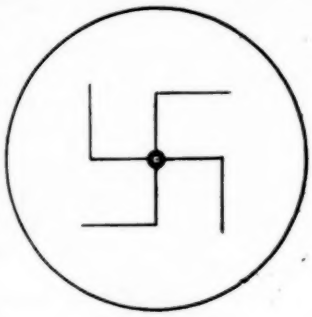

Fig. 3.

The galvanometer was of the ordinary D'Arsonval type. One centimeter deflection at a distance of 2 meters corresponded to 1.9×10^{-8} amperes.

A constant source of potential was obtained from small storage batteries. 10,000 volts were available. Without this constant source this investigation could not have been successfully carried out, especially at the lower pressures, where a variation of a few volts produced a large variation in the current.

The equation of the couple is

$$Fa = T_0\theta, \tag{2}$$

where F is the reaction on the point due to the ions in the gas streaming from it, a is the couple arm, T_0 the torsion constant and θ the angle of twist in radians. Substituting this in equation (1) we have

$$\theta = \frac{ai}{T_0K}\int dz. \tag{3}$$

Observations were made in the following way. The zero position of the couple was first observed with a telescope and scale. A potential was then applied to the points and the torsion head turned to keep the couple in the zero position. In this way the potential was varied until the desired current was obtained. The values of the current and the torsion were then noted. Owing to the fact that the current, the reaction, the distance between the points and the planes and the potential all varied simultaneously, it was not easy to duplicate conditions in consecutive readings. However, the results of these observations seldom differed among themselves by more than two or three per cent.

If a charge q is placed at a distance d from an infinite plane AB maintained at zero potential the density of the induced charge on

the surface of AB is inversely proportional to the cube of the distance, *i. e.*, $\sigma = \alpha/r^3$.

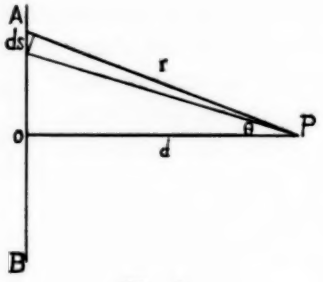

Fig. A.

Correction for electrostatic attraction between the point and plane.

The total charge on the plane will be

$$\int \sigma ds \qquad (a)$$

but

$$ds = r^2 \frac{dw}{\cos\theta}.$$

From (*a*) we get

$$\int_0^{2\pi} \frac{\alpha dw}{d} = \frac{2\pi\alpha}{d} = q,$$

$$\therefore \alpha = \frac{qd}{2\pi} \quad \text{and} \quad \sigma = \frac{qd}{2\pi r^3}.$$

Now the pressure per sq. cm. on the plane is

$$2\pi\sigma^2 = \frac{q^2d^2}{2\pi r^6}.$$

The total pressure

$$F = \int p ds = \frac{q^2d^2}{2\pi} \int \frac{ds}{r^6},$$

$$q^2 \int_0^{\pi/2} \frac{\cos^3\theta \sin\theta d\theta}{d^2} = \frac{1}{4}\frac{q^2}{d^2}.$$

Since $q = CV$, the above equation becomes

$$F = \frac{1}{4}\frac{C^2}{d^2} V^2, \qquad (4)$$

which gives the relation of the attraction between point and plane in terms of the applied potential and the distance. All the observations on the reaction produced on the point by a discharge from it had to be corrected for this force, which of course was of opposite sign to the force

of reaction. The method of making this correction was to observe the attraction between point and plane, using equation (2), as the potential was increased up to the point where a current began to flow. These attractive forces F were plotted with F as ordinates and PD as abscissas and the equation of the experimental curve determined in the following way. Assuming that the equation of this curve was of the form of equation (4), viz.,

$$v^n = PF,$$

then

$$n \log V = \log P + \log F,$$

replacing log V, log P and log F by $V'P'$ and F' we have

$$F' - nV' + P' = 0.$$

Since F' and V' are numbers obtained from the experimental curve they can be plotted. The result should be a straight line. From this

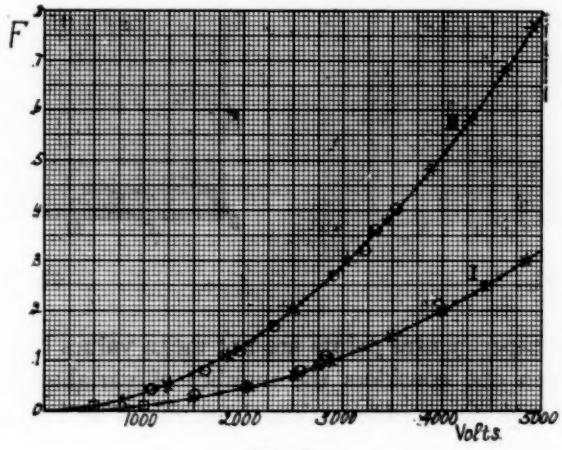

Fig. 4.

line n and P can be found. Fig. 4 shows typical curves of this kind obtained for different couples. The equation of the first is

$$V^{2.060} = 1.3 \times 10^8 F,$$

the second,

$$V^{1.971} = 2.47 \times 10^7 F.$$

Circles indicate observed points and crosses points calculated from the equations of the curves. The equations of a large number of these curves were determined and the values of the exponent of V were all within the limits of those given in the above equations. By extrapolating on these curves the values for the correction at the higher potentials were obtained.

RESULTS AT ATMOSPHERIC PRESSURE.

The curves in Figs. 5 and 6 were found by first determining the equation for the attraction at the different values of z and then measuring the reaction at these points. Fig. 5 shows the results obtained when i is constant and z varied. Fig. 6, the results when z is constant and i

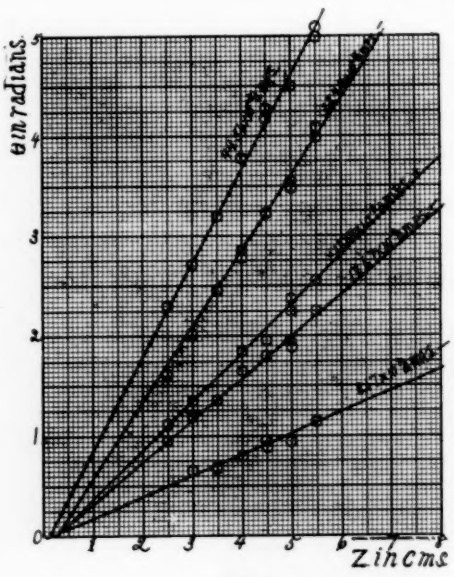

Fig. 5.

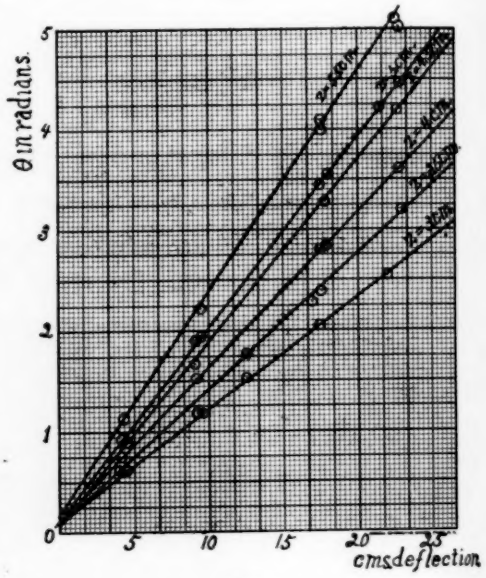

Fig. 6.

varied. The experimental points are not mean values but single observations. When one considers the number of variables that must be controlled in making an observation these points lie fairly close to the straight line which equation (3) would require.

According to equation (3), therefore, the slopes of the lines in Fig. 5 should equal ai/T_0K. Since we know all these quantities K can be calculated at once. However, when this was tried it was found that the value of K increased with the current. The slopes of the lines in Fig. 5 were plotted with the current and the straight line of Fig. 7 was obtained. The slope of this line is a/KT_0. This gives a constant value for K and indicates that for this experiment equation (1) should be written

$$F = \left(\frac{i}{K} + C\right) \int dz, \tag{5}$$

where C is a small constant.

K computed in this way came out 9.72 cm. for the negative ions and

6.43 cm. for the positive ions. Chattock[1] by measuring the wind pressure produced by electricity discharging from a point obtained $K_- = 1.80$ cm. and $K_+ = 1.32$ cm. Zeleny's[2] values were $K_- = 1.87$ cm. and $K_+ = 1.36$ cm. Franck[3] found K_- to vary from 8.8 to 12.26 cm. and K_+ from 2.5 to 3.77 cm. when he measured his velocities over short distances close to the point in strong fields using Zeleny's method. However, when he measured the velocities over greater distances using

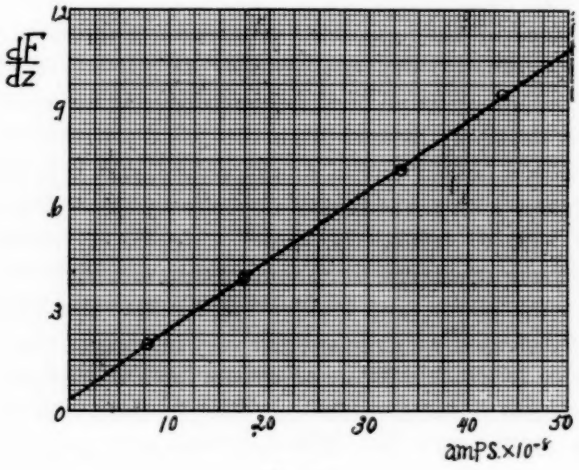

Fig. 7.

a modification of Rutherford's method, producing his ions from point discharge, he obtained $K_- = 1.79$ cm. and K_+ 1.34 cm., values which are in good agreement with those obtained by other observers.

If we consider the ratio, K_-/K_+, Zeleny's[4] value is 1.37, Chattock's[5] 1.36, Franck's[6] 1.35. This work gives 1.51 or taking the inverse ratio of the reactions from the data given in the second part of this paper $K_-/K_+ = 1.49$. The average of a number of observations made using the form of the apparatus of Fig. 3 gives 1.485. If we take Kovarik's[7] value for the velocity of the negative ion and the average of the above values for the positive we get for this ratio 1.507. Thus it appears that the ratios of the mobilities obtained from the reaction effects are in fair accord with those given by other methods, while the absolute values are very much too high. The following considerations may in

[1] Chattock, Phil. Mag., Series 5, 48, 1899.
[2] Zeleny, Phil. Trans. A., Vol. 195, p. 193, 1900.
[3] Franck, Ann. d. Phys., 4, Vol. 21, p. 972, 1906.
[4] Loc. cit.
[5] Loc. cit.
[6] Loc. cit.
[7] Kovarik, PHYS. REV., Vol. 30, p. 415, 1910.

part account for these discrepancies. Franck's work indicates that the velocity is very much higher near the point than it is farther out. Tyndall[1] concludes that whatever clustering there is, in air at atmospheric pressure, occurs immediately after the formation of the electron, *i. e.*, the electron does not travel an appreciable distance before it becomes attached. Now, if the ionization in the case of the negative begins farther from the point than in the case of the positive, and this is a most reasonable assumption, the method used by Franck would give a large difference in the apparent velocities. The mobility as calculated from equation (5) depends upon dF/dz. If this is cut down in any way the apparent value of K is increased. If we assume that the gas is ionized for some distance from the point and that the region of ionization extends farther out as z is increased, which is reasonable, as the field increases with z, the ions of opposite sign traveling back to the point would tend to reduce the value of dF/dz. If we assume also, that there is back discharge, *i. e.*, ions of opposite sign traveling from the planes to the points, and assume further that this back discharge increases as z increases, we have another very important factor in cutting down the value of dF/dz. Many observers have found back discharge, and Chattock and Tyndall[2] point out that in their experiments it increased with z. Zeleny[3] finds the back discharge greater for positive ions traveling from point to plane than for negative. If this is a correct view of the mechanism of point discharge large values of K would be expected from this method.

Another possible effect upon the reaction might be a forward drag upon the points due to the air currents set up in the gas by the moving ions. Such a drag, if it existed, would be directly proportional to the area of the points. Points whose area varied in the ratio 1, 2, 3 and 4 were used. The results showed that if such an effect existed it was negligibly small. This also tends to confirm the view that ionization begins at some distance from the point.

THE VARIATION OF MOBILITY WITH PRESSURE.

A number of experimenters have worked on this problem and have reached conclusions very much at variance with one another. Two methods have been used for this work. Rutherford[4] made the first observations in 1898, using the alternating field method. This method lends itself to measurements on the negative ion only. Rutherford

[1] Tyndall, Phil. Mag., S. 6, Vol. 21, May, 1911.

[2] Chattock and Tyndall, Phil. Mag., S. 6, Vol. 19, p. 455, 1910.

[3] Zeleny, PHYS. REV., Vol. 33, p. 70, 1911.

[4] Rutherford, Proc. of Camb. Phil. Soc., IX., p. 401, 1898.

concluded, as a result of his work, that the mobility of a negative ion was inversely proportional to the pressure. He used pressure as low as 34 mm.

Langevin[1] devised a method which enabled him to measure the mobility of both positive and negative ions with varying pressure. His observations extended over a range of pressures from 142 cm. down to 7.5 cm. of Hg. His results indicate a marked increase in the mobility of the negative ion beginning in the neighborhood of 20 cm. of Hg., the positive showing only a tendency to increase at 10 cm. of Hg. These observations were made in air, as were also Rutherford's.

Wellisch[2] in the Cavendish Laboratory, using Langevin's method, measured the mobility of both positive and negative ions in a large number of gases and vapors. His values for the mobilities at normal pressure are in good agreement with the results obtained by other observers. He found, however, that the mobilities of both positive and negative ions were inversely proportional to the pressures. The range extended from atmospheric pressure to 1 cm. of Hg.

Kovarik[3] used a slight modification of Rutherford's alternating field method. He worked with air and CO_2 and his range of pressures extended from 760 mm. to 8.8 mm. The method limited him to observations on the negative ion and his results show a very rapid increase in the mobility at about 10 cm. pressure. This rapid increase in the velocity of the negative ion at the lower pressures has always been interpreted to mean that the negative ion at ordinary pressure is an electron with a cluster of one or more molecules attached to it. At the lower pressures according to the results of Langevin and Kovarik it simplified its character by freeing itself from some or all of the attendant molecules, thus being able to move through the gas at a very much higher velocity, whereas the results of Rutherford and Wellisch seem to indicate no such simplification. Langevin's observations on the positive indicate a tendency toward simplification below 10 cm. while Wellisch finds that it appears to persist in its original form at pressures as low as 1 cm.

In view of the lack of complete agreement in the results obtained by the methods heretofore employed for measuring the mobility with varying pressure further experiments along this line seem desirable. Notwithstanding the unsuitability of the present method to the problem of a correct determination of the absolute value of K, the above observations would seem to indicate that it would probably yield consistent values

[1] Langevin, Ann. de Chemie et de Physique, t. XXVIII., p. 289, 1903.
[2] Wellisch, Phil. Trans., A., Vol. 209, pp. 249–279, 1909.
[3] Loc. cit.

for the variation of the mobility with varying pressure and if so, would be admirably adapted to throw light upon the disputed point regarding the change in the character of the ion at low pressure, since fairly accurate observations on the reaction could be obtained at lower pressures and in stronger fields than those at which it had been found possible thus far to measure mobilities. The results reached below tend to confirm this inference. Also observations made at 40 cm. pressure, when reduced to 76 cm. on the assumption that PK = constant, agreed well with values of K obtained at atmospheric pressure.

The arrangement of the apparatus was similar to that shown in Fig. 2. The pressure was measured with a mercury manometer which could easily be read to one tenth millimeter. The error in reading the pressure was well within the limits of the other errors of the experiment. The distance between the points and the planes was kept constant at 5.3 cm. Since it had been shown that for a given value of z, K is constant when i is constant, the observations were made with approximately constant currents and the value of K was obtained as follows from (1) and (2):

$$\frac{T_0\theta}{a} = F = \frac{i}{k}\int dz, \quad \therefore \quad K = \frac{i}{\theta}\, x \text{ constant.}$$

Assuming the law $K_0P_0 = KP$, we get

$$K_0 = \frac{Pi}{P_0\theta}\, x \text{ constant.}$$

The values of K_0 obtained in this way were plotted as abscissas with P as ordinates. If the mobility is inversely proportional to the pressure this should give a straight line.

Observations were made in air, hydrogen and CO_2. The pressure was varied from 760 mm. down to 5 mm. of Hg. The gases used were first thoroughly dried by passing them over calcium chloride and phosphorus pentoxide. Commercial CO_2 was used. The hydrogen was obtained by decomposing zinc with HCl in a Kipp generator. In order to remove oxygen and other impurities it was passed through pyrogalic acid and KOH, then through the drying tubes and finally admitted to the observation apparatus. The usual precautions were taken to free the apparatus from traces of air when filling it with the gas to be used.

Results at Reduced Pressures.

Figs. 8, 9 and 10 show the type of variation obtained. The negative curves in air and CO_2 begin to show an increase in the mobility of the ion, over that which the law PK = constant would require, at about 20 cm. pressure. At 10 cm. there is a decided bend in the curve and as the

pressure is still farther reduced the mobility increases enormously. The curves for hydrogen show the same characteristics except that the increase begins at a higher pressure and the decided bend occurs at about

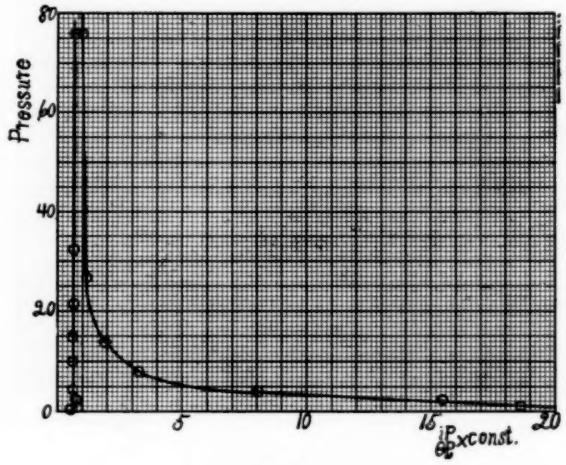

Fig. 8.

Air.

20 cm. Very high velocities are reached by the time the pressure is reduced to 8 cm. The hydrogen curve indicates a tendency for the

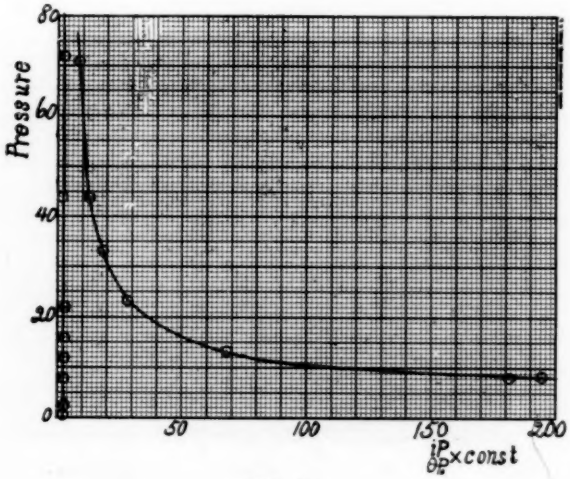

Fig. 9.

Hydrogen.

mobility to increase from the beginning. This may be due to the back discharge[1] which is so prominent in hydrogen.

[1] Chattock and Tyndall, Phil. Mag., Series 6, Vol. 19, p. 454, 1910; Tyndall, Phil. Mag., S. 6, Vol. 21, p. 585, 1911.

Fig. 11 shows the kind of accuracy that can be obtained from this method of measurement. The observations for the points marked with

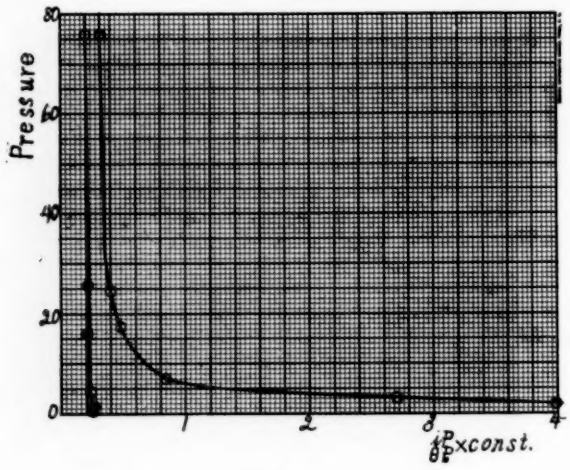

Fig. 10.

CO_2.

crosses were made three or four days after those for the points marked with circles. In the meantime the apparatus was repaired and refilled with CO_2.

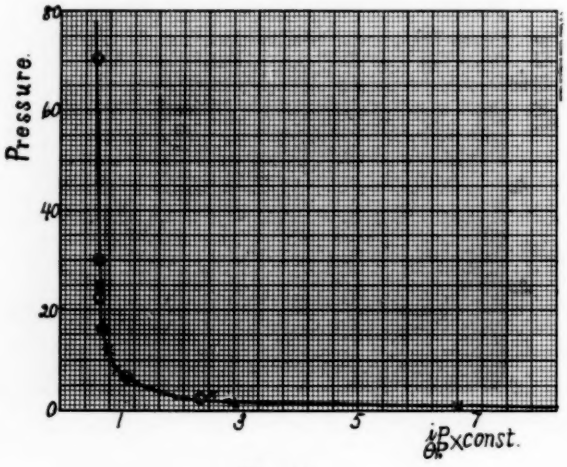

Fig. 11.

CO_2.

Tables I., II. and III. will serve to indicate the character of the data obtained by these observations.

TABLE I.

AIR.

Positive.

i Amperes.	P.D. Volts.	θ Observed.	θ Corrected.	P Cms.	$\frac{iP}{\theta P_0} \times$ Const.
12.7×10^{-7}	4,560	5.883	6.139	32.53	.749
12.0×10^{-7}	3,700	3.839	4.007	21.17	.716
12.3×10^{-7}	3,300	2.858	2.997	14.95	.684
12.3×10^{-7}	2,870	1.857	1.958	10.09	.709
12.6×10^{-7}	2,500	1.337	1.387	7.19	.729
12.3×10^{-7}	1,970	.658	.705	4.01	.779
12.2×10^{-7}	1,500	.303	.331	2.01	.797
12.2×10^{-7}	1,170	.125	.142	.85	.820
12.5×10^{-7}	1,070	.088	.092	.52	.794
		Negative.			
10.5×10^{-7}	4,000	4.533	4.729	40.70	1.019
10.15×10^{-7}	3,100	2.525	2.643	27.00	1.164
10.7×10^{-7}	2,190	.867	.926	14.18	1.855
11.25×10^{-7}	1,490	.274	.301	8.01	3.34
11.8×10^{-7}	1,000	.0573	.0698	4.26	8.06
10.7×10^{-7}	725	.0112	.0167	2.17	15.56
10.0×10^{-7}	720	.00101	.0065	1.12	18.75

TABLE II.

HYDROGEN.

Negative.

i Amperes.	P.D. Volts.	θ Observed.	θ Corrected.	P Cms.	$\frac{iP}{\theta P_0} \times$ Const.
21.6×10^{-7}	2,940	1.975	2.081	71.28	8.29
22.6×10^{-7}	1,960	.736	.883	44.72	12.39
22.6×10^{-7}	1,580	.4084	.439	33.43	19.22
22.2×10^{-7}	1,180	.195	.213	23.88	28.11
23.8×10^{-7}	890	.0426	.0524	13.51	68.70
22.6×10^{-7}	800	.0382	.0462	8.27	181.3
		Positive.			
				71.70	2.30[1]
12.05×10^{-7}	3,160	2.500	2.622	44.53	2.28
12.4×10^{-7}	2,801	1.705	1.801	31.42	2.41
12.2×10^{-7}	2,430	1.186	1.259	22.26	2.42
12.2×10^{-7}	2,140	.8238	.899	16.54	2.51
11.8×10^{-7}	1,880	.500	.543	12.08	2.91
12.2×10^{-7}	1,560	.359	.389	8.10	2.82
12.4×10^{-7}	1,250	.188	.208	4.63	3.15
12.7×10^{-7}	980	.0875	.0995	2.18	3.02
12.5×10^{-7}	815	.0419	.0489	.89	2.63
12.9×10^{-7}	823	.0274	.0345	.51	2.14

[1] Calculated.

TABLE III.

CARBON DIOXIDE.

Negative.

i Amperes.	P.D. Volts.	θ Observed.	θ Corrected.	P Cms.	$\frac{iP}{\theta P_0} \times$ Const.
7.9×10^{-7}	——	——	16.0	70.4	.387
7.9×10^{-7}	3,800	5.035	5.21	24.48	.407
7.9×10^{-7}	3,280	3.167	3.29	17.69	.473
8.2×10^{-7}	2,000	.7365	.785	7.15	.830
8.6×10^{-7}	1,230	.1012	.1202	3.43	2.74
6.8×10^{-7}	800	.0101	.0109	.95	4.00
		Positive.			
4.3×10^{-7}	4,760	5.21	5.44	25.38	.225
5.6×10^{-7}	3,500	2.959	3.009	10.8	.226
5.5×10^{-7}	2,380	1.033	1.102	4.40	.246
4.0×10^{-7}	1,750	.2304	.267	1.53	.258
5.6×10^{-7}	1,550	.063	.092	.41	.268

From these results it seems clear that whether the negative ion is a charged molecule (Wellisch's view) or a cluster of molecules (Langevin's view) it simplifies its character in the neighborhood of 10 cm. pressure for air and CO_2 and 20 cm. for hydrogen; on the other hand, whatever the character of the positive ion it remains unchanged at the lowest pressures observed.

Since this work was completed (September, 1910), and presented to the American Physical Society (December, 1910), a paper has appeared by Todd[1] which gives the results which he has obtained by so modifying Rutherford's method as to make it possible to use it in measuring the mobility of positive ions at reduced pressure.

Todd's results are in complete agreement with those found above, in that the law PK = constant holds for positive ions at pressures as low as 1 mm.

CONCLUSIONS.

1. The relation between the force of reaction and the distance between point and plane in point discharge has been investigated and found to be linear within the range of currents and distances used. The relation between the force and the current for different values of the distance was also found to be linear. The experimental relation is expressed by the equation

$$F = \left(\frac{i}{K} + C\right)\int dz.$$

[1] Camb. Phil. Soc. Proc., 16, p. 21, February, 1911.

2. The absolute values of the velocities of the ions in unit field were found to be very much higher than those obtained by other methods. The ratio of the velocities, however, was only slightly higher. A possible explanation of the high velocities is suggested.

3. The variation of the mobility with pressure has been measured from 760 mm. to 5 mm. pressure in air, hydrogen and CO_2. The mobility of the positive was found to be inversely proportional to the pressure in all cases down to 5 mm. The negative ion showed a rapid increase in mobility, beyond that required by the law PK = constant, beginning at about 20 cm. Below 10 cm. the mobility became very large. In the case of hydrogen these effects occur at somewhat higher pressures.

Before concluding the author wishes to thank Professor A. A. Michelson and the members of the Ryerson staff for their interest and encouragement, especially Professor R. A. Millikan, at whose suggestion this investigation was undertaken, and who has offered many helpful suggestions during its progress.

RYERSON LABORATORY,
UNIVERSITY OF CHICAGO,
August, 1911.

THE MAGNETIC ROTATION AND ELLIPTICITY PRODUCED BY MIRRORS OF MASSIVE METALS.

BY PAUL D. FOOTE.

KERR,[1] Righi[2] and others have shown that plane polarized light, incident at any angle upon a mirror of a magnetic substance placed in a magnetic field, experiences certain modifications upon reflection. In general the reflected vibration takes the form of an ellipse the axes of which are inclined at an angle to the plane of the incident vibration. The magnitudes of this ellipticity and rotation vary with the intensity of magnetization, the angle which it makes with the normal to the mirror, the wave-length of the light, the angle of incidence, and the material of which the mirror is constructed. The more simple case of normal magnetization and normal incidence of the light is of especial interest because the effect is not then complicated by the presence of phenomena arising from ordinary metallic reflection.

Rotation.—DuBois[3] has observed a direct proportionality between the rotation (R) and the intensity of magnetization (I) normal to the reflecting surface; that is,

$$R = K \cdot I,$$

where K is known as the "Kerr constant" for the given substance and wave-length. For iron, cobalt, and nickel, he found K to be negative (the rotation being contrary to the direction of the magnetizing current) while in the case of magnetite K was positive. The dispersion of the rotation was also investigated—iron showing a linear increase of the Kerr constant with increasing wave-length; nickel presenting a minimum at $580\mu\mu$; cobalt a slight minimum in the blue-green; and magnetite a distinct maximum at $590\mu\mu$.

Ingersoll[4] working in the infra-red has extended DuBois' curves to a wave-length of 4μ and found that beyond a wave-length of 1μ K decreases for the magnetic metals and magnetite. The rotatory dispersion curves obtained by him resemble typical dispersion curves in the region of an

[1] J. Kerr, Phil. Mag. (5), 3, p. 339, 1877; (5), 5, p. 161, 1878.
[2] A. Righi, Ann. de Chim. et Phys. (6), 4, p. 433, 1885; (6), 9, p. 132, 1886.
[3] H. E. J. G. DuBois, Wied. Ann., 39, p. 25, 1890.
[4] L. R. Ingersoll, Phil. Mag., p. 41, Jan., 1906.

absorption band. In the case of magnetite and nickel the rotation changes sign in the neighborhood of 1μ. Heusler's alloy was found to give, if any effect at all, at most one tenth that of iron or steel. Measurements[1] made upon iron mirrors deposited by a cathode discharge gave curves similar to those from massive steel.

Loria[2] has added several substances to the list of those which exhibit the Kerr effect. The rotation in the case of magnetite he found to change sign at $464\mu\mu$. With invar it increased with increasing wave-length up to $600\mu\mu$ where a slight negative maximum occurred. Cupriferrite showed a positive rotation up to $587\mu\mu$ where it passed through the zero and finally reached a slight maximum negative rotation at about $650\mu\mu$. The magnitude of all these rotations was small—the maximum for the saturated cupriferrite being approximately but one fifteenth of the maximum for saturated iron as observed by DuBois.

Tokmatschew[3] predicted a rotation in saturated Heusler's alloy of about $14'$ for $450\mu\mu$, but neither Loria working in this region of the spectrum, nor Ingersoll in the infra-red, discovered any effect.

Ellipticity.—Comparatively little work has been done on the ellipticity which generally is present with rotation. The first measurements of this, at normal incidence and normal magnetization, were made by Zeeman,[4] who investigated the so-called Sissingh constant for iron and cobalt at a wave-length of $589\mu\mu$.

Skinner and Tool[5] have also investigated, incidentally, both rotation and ellipticity in the case of iron, cobalt and nickel cathodic films used as mirrors and found them when present at all to be of the same sign as for the massive metals, that is, in a direction opposite to the magnetizing current. In their measurements the ratio of the axes of the reflected elliptic vibration was of an order 3×10^{-3} and approximately the same for all wave-lengths. Practically no ellipticity could be observed in light reflected from films of cobalt either electrolytic or cathodic, nor were cathodic films of nickel "active" even in transmitted light.

The present work is a report of an investigation of primarily the ellipticity, but along with it also the rotation, produced in plane polarized light incident normally upon mirrors of massive iron, cobalt, nickel, steel, magnetite, invar, and Heusler's alloy, all magnetized normally to the re-

[1] L. R. Ingersoll, Phil. Mag., p. 74, July, 1909.

[2] Stanislaw Loria, Koninklijke Ak. v. Wet. te Ams., May, 1910.

[3] S. Tokmatschew, Journ. de russ. phys.-chem. Ges., 42 (Phys. T.), 1910. Beibl., N. 13, p. 718, Art. 51, states that this rotation was *observed*, while Loria claims it to be *predicted* from theoretical considerations. The author was unable to obtain the original article.

[4] Dr. P. Zeeman, Leiden Comm. No. 15, 1895.

[5] C. A. Skinner and A. Q. Tool, Phil. Mag., p. 833, Dec., 1908.

flecting surface. In the case of iron, nickel invar and magnetite the refractive indices and absorption coefficients of the same surfaces were also measured. These were obtained by Mr. J. T. Tate[1] of this laboratory, using the method of measurement described by Tool.[2] In order to preclude the possibility of any time-change in the surfaces affecting the results his observations were made as soon as those on the rotation and ellipticity were completed.

Methods of Measurement.

Rotation.—Figure 1 shows diagrammatically the manner in which the apparatus was arranged. For the measurement of rotation, bright sunlight was concentrated upon the slit of a Fuess spectral system and passing successively through a lens L_2, a total reflecting prism, and polarizer, was reflected from the tested mirror M, and brought to a focus

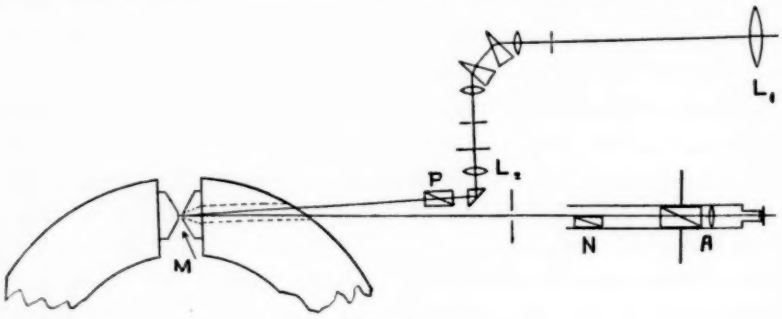

Fig. 1.

upon the Lippich half nicol N set at a small angle to the analyzer A. The angle between the incident and reflected ray was about 2°—well within the limit of 15° for which, as has been shown by Righi, the rotation is practically the same as for normal incidence. The spectral system was so adjusted that it gave a field of light ranging in wave-length over 20$\mu\mu$ in the red and 15$\mu\mu$ in the violet. Inasmuch as there were no rapid variations of the observed effects with wave-length this gave a sufficiently homogeneous light.

Ellipticity.—For the measurements of ellipticity, a Brace halfshade and compensator analyzing system was used. In the place of the Lippich N, was substituted the fixed halfshade on which a telescope was focused. Between this and the analyzing nicol, was mounted the compensator. To compensate for the rotation of the plane of polarization, before attempting to measure the ellipticity, the polarizer was provided with a

[1] The author wishes to acknowledge his indebtedness to Mr. Tate.

[2] Phys. Rev., page 1, July, 1910.

means for being slightly rotated so that in all cases the axes of the elliptic vibration could be set at 45° to the azimuth of the fixed halfshade—this being the condition of maximum sensibility. The order of the compensator which was carefully measured by Mr. Tate[1] was 6.89° for a wave-length of 550$\mu\mu$. The order for other wave-lengths was calculated from the data obtained by Tool with similar mica compensators. The order of the halfshade was about 3°. A slight modification of the general formula by Professor Tuckerman[2] of this laboratory for such an arrangement of the analyzing system is

$$E = \tan N_1[\sin 2(\varphi - \varphi_0) + \sin 2\varphi_0],$$

in which E = ratio of minor to major axis of the ellipse; $2N_1$ = order of the compensator; φ_0 = angle through which the compensator is rotated to change from a match on plane polarized light to the nearest position of no effect on this light; and φ = the angular displacement of the compensator from its position of match on plane polarized light to a match on light of ellipticity E.

All measurements of both ellipticity and rotation were obtained by reversing the magnetizing current and thus doubling the optical effect.

The direction of the vibration in the ellipse was determined by comparing the compensator with a quarter wave plate, of which the "fast" and "slow" azimuths had been determined by means of the interferometer.

Accuracy of the Optical Measurements.—The probable error of the observations varied with the conditions of the sunlight and the number of individual settings made. In the case of rotation it usually amounted to ± .005°. Similarly the error in ellipticity was approximately ± .05 $\times 10^{-3}$. When the observed magnitudes were very small, a larger number of readings, frequently from 20 to 30, were taken and the probable error of the mean thereby considerably reduced. Added to the above error in the ellipticity, due to the compensator setting, would be any error in the calibration of the compensator plate. A liberal allowance for this is one per cent.

Mirror Polish.—The polishing of the mirrors especially in the case of the softer metals such as cobalt, nickel, invar, or Heusler's alloy was very troublesome. However a satisfactory surface free from scratches could be obtained by the use of soft "silver rouge" on a bed of asphaltum. In a number of cases mirrors were polished with rouge or tin oxide upon a bed of pitch. These latter were plane and gave an excellent image but

[1] Using a combination of Stokes's method and the halfshade principle, Tool, loc. cit.

[2] The Transmission of Light through Doubly Refracting Plates, with Applications to Elliptic Analyzing Systems. L. B. Tuckerman, Jr., University of Nebraska Studies, April, 1909.

were slightly scratched. Those polished on asphaltum were also fairly plane, but did not present the bright clear appearance of the others. Regarding the effect of methods of polishing upon the magneto-optical constants, one appears warranted in concluding that, as with the ordinary optical constants, the form of the curve is practically unchanged though the magnitude of the constants may differ somewhat. Great care was always necessary lest oils, benzol or any other cleaning agent, should dry upon the mirror surface, as the deposit from these substances can affect the constants to a marked degree.

Measurement of Field Strength.—The measurement of the strength of the magnetic field was made by means of the rotation produced by a slab of glass for which the Verdet constant had been carefully obtained One very important consideration seems to have been neglected by previous observers in this line of work. This is the variation of the field strength with the distance from the mirror surface. A thin cover glass was mounted at different distances from the pole piece by means of brass rings of various thicknesses. The magnetic rotation thus obtained when steel and magnetite mirrors were used is plotted in Fig. 2 against distances

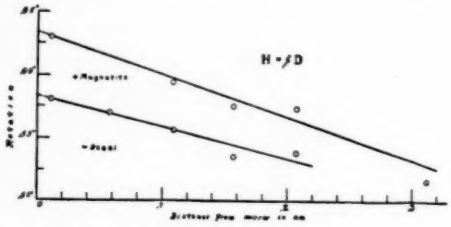

Fig. 2.

to the center of this cover glass. The curves for the other mirrors were similar. The variation with distance is marked but the relation seems to be approximately linear. When therefore a standard calibration slab is used the field measured is that at the center of the slab rather than at the surface of the mirror. The true value at the mirror was, in the present work, obtained by extrapolation. The largest correction necessary was about six per cent.

Experimental Results.

The measurements of rotation for all the mirrors are incorporated in Table I., and those of ellipticity in Table II. In Table III. are given the ordinary optical constants of several, in which as commonly used, ν_0 is the refractive index for normal incidence and κ_0 the extinction coefficient —being the ratio of the amplitude at any point to that at a distance

farther ahead (in the direction of propagation) equal to one wave-length in the substance.

Iron.

The curves representing the variation with wave-length of both rotation and ellipticity for the various iron mirrors are given in Fig. 3.

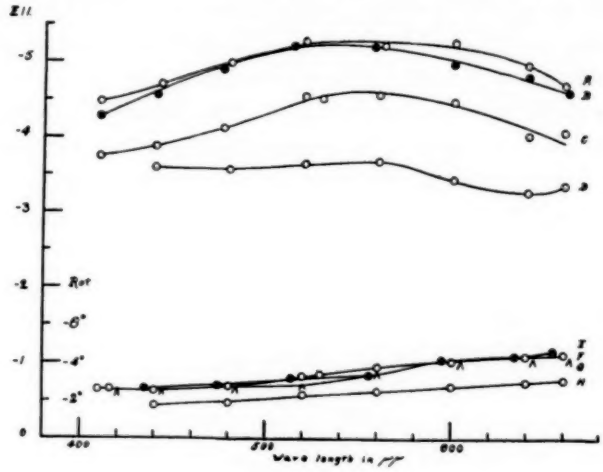

Fig. 3.

Iron.

A,	Mirror 7,	$E \times 10^3$,	21,000 C.G.S.,	*E*, ●,	Mirror 4,	$R \times 10$,	20,400 C.G.S.,
B,	Mirror 4,	$E \times 10^3$,	20,400 C.G.S.,	*F*, ○,	Mirror 8,	$R \times 10$,	22,900 C.G.S.,
C,	Mirror 8,	$E \times 10^3$,	22,900 C.G.S.,	*G*, ∧,	Mirror 7,	$R \times 10$,	21,000 C.G.S.,
D,	Mirror 1,	$E \times 10^3$,	13,600 C.G.S.,	*H*,	Mirror 1,	$R \times 10$,	13,500 C.G.S.

Commercial Swedish Iron Mirror 1.—Polished with rouge on a bed of pitch. Field strength, 13,600 C.G.S. The rotation is negative throughout the spectrum, the magnitude increasing with increasing wave-length. The ellipticity is also negative with a slight maximum value at 560$\mu\mu$. The rotation is about twice the magnitude of that obtained by Skinner and Tool, with cathodic films of iron in a slightly weaker field (13,200), while the ellipticity is about 70 per cent. greater than their values. The effects with this mirror were not maxima since the magnetic saturation was not reached.

Commercial Swedish Iron Mirror 4.—Polished two years before using —surface bright and clear. Field strength, 20,400. The rotatory dispersion and ellipticity curves of this are similar in form to those obtained for Mirror 1 but due to the stronger field are somewhat greater in magnitude.

Pure Wrought Iron Mirror 7.—This iron was obtained from Kahlbaum, somewhat flawed, but admitted a satisfactory polish with pitch and

rouge. Field strength, 21,000. The rotation by this mirror increased with increasing wave-length, but not exactly linearly. The ellipticity curve follows closely that obtained for Mirror 4.

Pure Wrought Iron Mirror 8.—From same sample as 7. Field strength, 22,900. The rotation increased with wave-length, changing from a slight convexity to the axis of abscissas below 520$\mu\mu$ to a slight concavity above. The ellipticity curve resembles in form that of the preceding mirrors, but its magnitude is less than that of either Mirror 4 or 7.

Fig. 6 gives a graph of the variation of the rotation produced by this mirror with field strength. It has been shown by DuBois that the abscissa of the point of intersection of the straight line, $R = KH$, and the asymptote, R = constant, has the value $4\pi I_m$, where I_m is the saturated intensity of magnetization. Accordingly for pure iron (Mirror 8) we find from the figure the intersection at about 19,100, hence I_m = 1,520. Fig. 6 also shows for the same mirror a plot of the ellipticity as a function of field strength. The curve is very similar to that of rotation.

If the values of the intensity of magnetization, obtained from the rotation curve by the aid of the equation $R = KI$, be plotted against ellipticity (Fig. 8), we see at once a direct proportionality exists, so that we may write for the ellipticity a similar equation

$$E = FI,$$

where F is the proportionality constant of the mirror for any given wave-length. That the same relation exists for transmitted light has already been shown by Skinner and Tool (l. c.). The ordinary optical constants of this mirror are given in Table III.

COBALT.

The cobalt was obtained from Kahlbaum in rolled sheets 2 mm. thick. The curves from the various samples are plotted in Fig. 4. The rotation for all is negative, with a very slight minimum in the yellow. The ellipticity is also negative, the magnitude decreasing with increasing wave-length. In the blue all tend toward a maximum ellipticity and would probably show decreasing values in the ultra-violet. The general form of both sets of curves for the four mirrors are in agreement, but the magnitudes are somewhat different. Skinner and Tool obtained approximately the same rotation with cathodic films of cobalt but were unable to observe any ellipticity.

Fig. 6 shows the variation of rotation and ellipticity with the magnetic field. Both curves resemble typical I–H curves, and if the intensity of

magnetization, obtained from the rotation values, be plotted against ellipticity, the result is a straight line, as was the case with iron (Fig. 8). I_m is found from the above relation to be approximately 1,390 C.G.S.

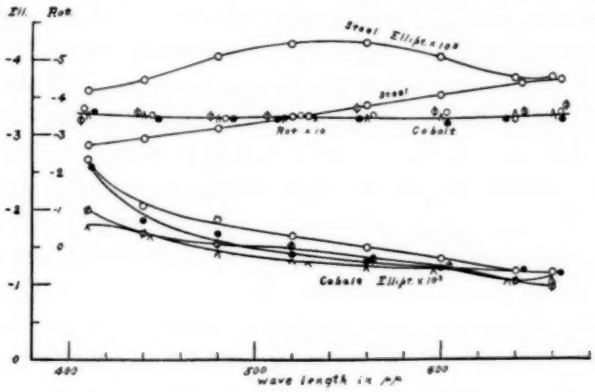

Fig. 4.

○ Cobalt Mirror 2, ⊖ Cobalt Mirror 5,
● Cobalt Mirror 3, ∧ Cobalt Mirror 10.

NICKEL.

The nickel was obtained from Kahlbaum also in the form of sheets 2 mm. thick.

Mirror 6.—This mirror was polished by buffing. The surface was

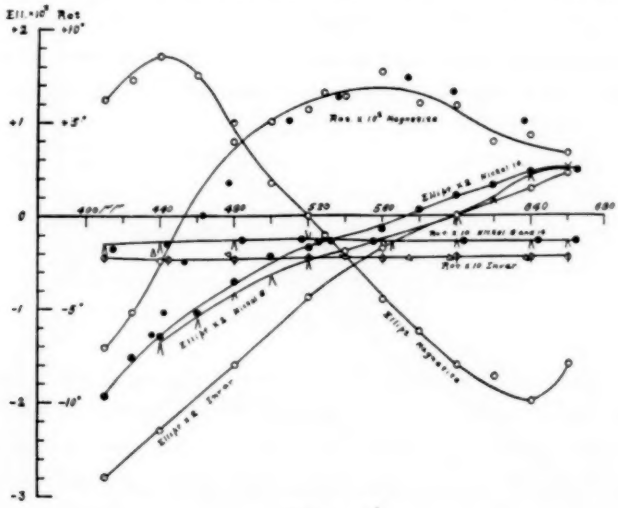

Fig. 5.

somewhat pitted and not exactly plane. Field strength, 17,800. The curves from it are found in Fig. 5. The rotation is negative with a slight minimum in the green. The ellipticity presents an interesting feature

in that it reverses sign in passing through the spectrum. In the violet, green, and yellow, it is negative, the magnitude decreasing gradually with increasing wave-length; at 600$\mu\mu$ it passes through the zero (*i. e.*, the light remains plane polarized upon reflection) and from this point to 660$\mu\mu$ continues positive with gradually increasing values.

Mirror 14.—Polished with rouge and tin oxide on a bed of pitch. The surface was plane, bright and clear, but showed a few fine scratches. Curves for this also in Fig. 5; field strength, 20,500. The rotation is negative and practically of the same magnitude as that obtained with Mirror 6. A very slight minimum probably exists at 520$\mu\mu$. The ellipticity reverses sign as in the case of the preceding mirror crossing the axis at 572$\mu\mu$ instead of at 600. This disagreement in the two may be due to the difference in the method of polishing or possibly to errors of observations, since the compensator rotation was very small, although to balance this the values for both curves were made the mean of from 20 to 30 observations. Fig. 7 presents a graph of the rotation as a function of the field strength. I_m is found to be 600 C.G.S.—less than half that of iron or cobalt. The ordinary optical constants of this mirror are given in Table III.

Steel.

One hardened steel mirror was studied. This was obtained from a Lissajous figure apparatus and possessed a large bright clear surface. The curves are found in Fig. 4—field strength, 19,200. They are similar in form and of approximately the magnitude of those from iron. Figure 6 shows the variation of rotation and ellipticity with the field. Both

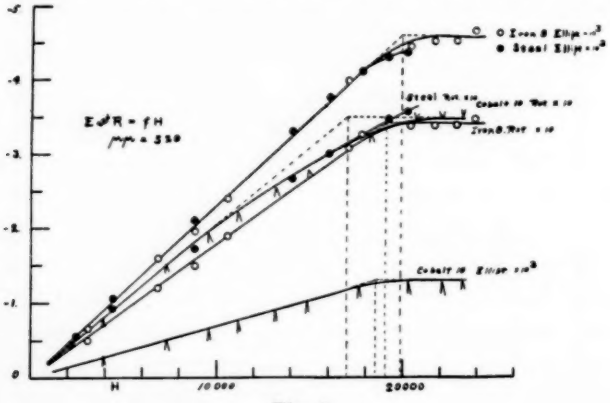

Fig. 6.

curves have the general *I–H* form, but, although the induction was brought up to 20,000, it is seen that the asymptotic sections of the curves are not quite reached. As was to be expected, the maximum intensity

of magnetization is accordingly higher than that for any other substance tested.

Invar.

The invar, containing 36 per cent. nickel and 64 per cent. iron, was obtained in the form of bars 3 × 12 × 250 mm. from the Haen chemical works, Seelze, Germany. An excellent polish was secured by means of silver rouge and asphaltum. The curves from this are given in Fig. 5. The rotation is negative and of a magnitude lying between that of nickel and iron, the curve bearing a closer similarity to the former. A slight maximum is reached at 480$\mu\mu$ but the general form is approximately a straight line of zero slope. The values given by Loria are represented in the plot by triangles. The agreement in values is fairly satisfactory. The ellipticity shows strikingly the effect of the presence of nickel. Precisely as with that of Nickel Mirror 6 the value is negative in the blue, gradually decreasing with increasing wave-length to the point 600$\mu\mu$ where it passes through the zero and becomes positive, with gradually increasing magnitude. The negative values from the alloy, however, are larger and the positive values smaller than those from the nickel, indicating the influence of the iron present. The rotation and ellipticity as functions of the field, shown in Fig. 7, present no new char-

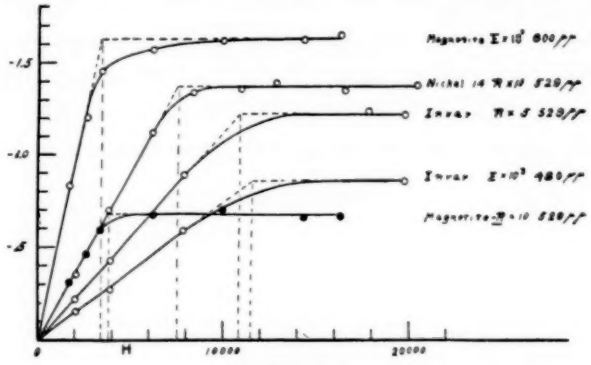

Fig. 7.

acteristics. The maximum intensity of magnetization obtained from a mean of the values given by each curve is 900 C.G.S.—much larger value than that found by Loria (530 C.G.S.). Fig. 8 shows the proportionality between ellipticity and intensity of magnetization. The ordinary optical constants for this invar mirror are found in Table III. They differ very little from those obtained from iron, scarcely showing any effect of the presence of nickel.

Magnetite.—(Ferroferrite, $FeO.Fe_2O_3$.) Octahedral crystal from Pfitschthal, Tyrol. A section 2 mm. thick was cut parallel to one of the faces

and polished with rouge and pitch. The surface was brilliant, plane and free from scratches. The rotatory dispersion is shown in Fig. 5. In the violet the rotation is negative, the magnitude rapidly increasing with decreasing wave-length. It is possible that a maximum value exists in the early part of the ultra-violet. The curve passes through the zero point at about $454\mu\mu$ and then continues positive throughout the visible spectrum until a maximum value is reached at $550\mu\mu$. It is interesting to note that Ingersoll[1] working in the infra-red found the curve to again pass through the zero at about 1μ and to continue negative in magnitude as far as investigated (4μ). The curve obtained by Loria (indicated by full dots) is of the same general form but appears to be shifted a small distance toward the red. DuBois[2] in 1890 using the crystal of magnetite which was later tested by Loria, obtained a curve shifted still farther toward the red.

The curve of ellipticity as a function of the wave-length is also of an interesting form. In the violet it is positive, a decided maximum existing at $440\mu\mu$. From here the magnitude decreases with increasing wave-length until the point $520\mu\mu$ is reached where the ellipticity is zero. As the wave-length still further increases, the magnitude of the negative ellipticity increases almost linearly up to the point $640\mu\mu$ where a sharp negative maximum exists.

In Fig. 7 are plotted the results of the measurements of both rotation and ellipticity with various field strengths. The saturated intensity of magnetization (I_m) is accordingly found to be 287 C.G.S. This value is considerably smaller than has heretofore been obtained with magnetite

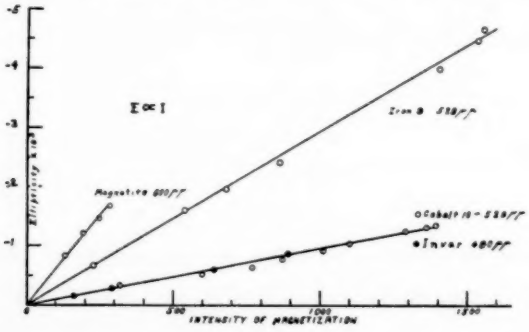

Fig. 8.

crystals. DuBois found the value 350, Loria 358, and Quittner[3] by an entirely different method about 480. However magnetite, as has been

[1] Ingersoll, loc. cit.

[2] DuBois, loc. cit.

[3] V. Quittner, Dissertation, Zurich, 1908.

shown by the latter investigator, is a substance which exhibits a great diversity of structure, the density and other properties varying with different samples of apparently the same crystalline form. It is possible, as has been suggested by Loria, that such phenomena have some influence upon the optical behavior of the material.

Fig. 8 shows the ellipticity as a function of the intensity of magnetization. In Table III. are found the ordinary optical constants of this mirror. As found by Skinner and Tool (l. c.) for cathodic films of iron, there appears to be nothing in these optical constants which suggests an explanation for the reversal of sign of the magneto-optical constants.

Heusler's Alloy.

This alloy was also secured from the Haen chemical works. It contained: Mn, 26 per cent.; Al, 13 per cent.; Cu, 61 per cent. The interior of the sample was full of bubbles, but a satisfactory slab 2 mm. thick was cut from the surface and made into several small mirrors which were polished with rouge on asphaltum. Practically no rotation could be detected in the violet, but a slight negative rotation was measured in the red. At 605$\mu\mu$ this amounted to $-.03°$ and at 640 to $-.05°$. The ellipticity was negative but of an order too small to justify measurements throughout the spectrum. At 440$\mu\mu$ a number of observations gave the magnitude as -3×10^{-4}. The effect was distinctly smaller in the red.

Table I.

Rotation (Degrees).

Mirror.	Field (C. G. S.).	410$\mu\mu$	440$\mu\mu$	480$\mu\mu$	520$\mu\mu$	560$\mu\mu$	600$\mu\mu$	640$\mu\mu$	660$\mu\mu$
Iron 1.....	13,600	———	—.17	—.19	—.23	—.25	—.28	—.30	—.31
Iron 4.....	20,400	———	—.26	—.28	—.32	—.34	—.42	—.44	—.47
Iron 7.....	21,000	—.25	—.26	—.28	—.29	—.36	—.42	—.43	—.44
Iron 8.....	22,900	—.250	—.253	—.277	—.329	—.379	—.413	—.443	—.449
Cobalt 2...	20,000	—.37	—.35	—.34	—.35	—.35	—.36	—.34	—.36
Cobalt 3...	20,000	—.36	—.34	—.34	—.34	—.34	—.33	—.34	—.34
Cobalt 5...	20,000	—.34	—.36	—.35	—.35	—.37	—.36	—.36	—.38
Cobalt 10..	22,100	—.362	—.363	—.352	—.353	—.354	—.358	—.367	—.368
Nickel 6...	17,800	—.15	—.15	—.13	—.13	—.15	—.13	—.14	—.14
Nickel 14..	20,500	—.181	—.147	—.129	—.129	—.138	—.144	—.137	—.140
Steel.....	19,200	—.270	—.285	—.313	—.348	—.378	—.405	—.438	—.445
Invar	19,800	—.219	—.234	—.235	—.231	—.230	—.222	—.227	—.227
Magnetite .	16,400	—.071	—.024	+.040	+.057	+.077	+.059	+.043	+.033

TABLE II.

Ellipticity $\times 10^3$.

Mirror.	Field (C. G. S.).	410μμ	440μμ	480μμ	520μμ	560μμ	600μμ	640μμ	660μμ
Iron 1.....	13,600	———	—3.60	—3.58	—3.66	—3.70	—3.47	—3.31	—3.40
Iron 4.....	20,400	—4.29	—4.58	—4.95	—5.27	—5.25	—5.02	—4.86	—4.65
Iron 7.....	21,000	—4.49	—4.67	—5.01	—5.30	—5.25	—5.30	—5.01	—4.75
Iron 8.....	22,900	—3.73	—3.88	———	—4.56	—4.60	—4.50	—4.06	—4.12
Cobalt 2...	20,000	—2.67	—2.04	—1.85	—1.63	—1.49	—1.33	—1.17	—1.15
Cobalt 3...	20,000	—2.61	—1.85	—1.67	—1.41	—1.31	—1.22	—1.20	—1.13
Cobalt 5...	20,000	—2.00	—1.68	—1.54	—1.52	—1.34	—1.27	—1.04	— .97
Cobalt 10..	22,100	—1.79	—1.68	—1.43	—1.33	—1.23	—1.20	—1.07	—1.08
Nickel 6...	17,800	———	— .68	— .41	— .25	— .12	—0.00	+ .21	+ .25
Nickel 14..	20,500	— .97	— .65	— .35	— .17	— .07	+ .11	+ .23	+ .24
Steel.....	19,200	—3.59	—3.73	—4.04	—4.21	—4.23	—4.03	—3.76	—3.77
Invar.....	19,800	—1.40	—1.15	— .80	— .44	— .17	—0.00	+ .14	+ .23
Magnetite..	14,400	+1.24	+1.71	+1.00	.00	— .91	—1.61	—2.00	—1.60

TABLE III.

	Iron Mirror 8.		Nickel Mirror 14.		Invar.		Magnetite.	
μμ	ν_0	κ_0	ν_0	κ_0	ν_0	0	ν_0	0
460	2.032	1.445	1.927	1.556	2.12	1.44	2.394	.314
520	2.210	1.395	1.942	1.732	2.28	1.37	2.402	.297
560	2.340	1.335	2.005	1.788	2.39	1.37	2.425	.282
600	2.437	1.305	2.082	1.820	2.50	1.37	2.451	.266
660	2.548	1.270	2.215	1.837	2.66	1.36	2.468	.241

SUMMARY.

1. The magnetic rotation and ellipticity by reflection at normal incidence have been measured throughout the visible spectrum for massive mirrors of iron, cobalt, nickel, steel, invar, magnetite, and Heusler's alloy.

2. The rotation is in reasonable agreement with the work of preceding investigators.

3. The ellipticity, the investigation of which was the main object of the present work, and which has not been studied extensively by other observers, was found to be consistent with what might be expected from the work which has been done on transmission. For the majority of substances tested it was negative throughout; but for others it changed sign within in the visible spectrum.

4. The ordinary optical constants obtained for iron, nickel, invar and magnetite showed no evident connection existing between these and the magneto-optical constants.

5. In a number of cases, careful attempts to reproduce the mirror polish failed to give results which agreed in magnitude though the curves were usually of the same general form. In others practically identical results were obtained, although the mirrors were subjected to different methods of polish. That the polish may affect the magnitude and not the form of the dispersion curves has been quite generally observed by investigators of the ordinary optical constants of mirrors.

6. As with transmitted light the ellipticity of the reflected light in every case was found proportional to the intensity of magnetization for all wave-lengths.

In conclusion the writer wishes to thank Professors C. A. Skinner and L. B. Tuckerman for the aid he has received in this experiment.

THE BRACE LABORATORY OF PHYSICS,
THE UNIVERSITY OF NEBRASKA,
July, 1911.

A STUDY OF THE REVERSIBLE PENDULUM.[1]

PART II. EXPERIMENTAL VERIFICATIONS.

BY JOHN C. SHEDD, J. A. and W. N. BIRCHBY.

FOR each axis of suspension of a physical pendulum there are, in general, three other parallel axes about which the pendulum swings with the same period. As shown in Fig. 1, these axes are symmetrically located in pairs with respect to the centroid of the pendulum. If z' and z'' be the distances of the respective pairs from the centroid, they are connected by the well-known relation

$$z'z'' = a^2,$$

in which a is the radius of gyration of the whole pendulum about a line passing through its centroid and parallel to the above axes.

If now the position of the centroid be varied (and with it the value of a) by altering the distribution of mass of the pendulum, the variation of z' is expressed by equation (9), as a linear function of d. The variation of z'' is more complex and may be derived as follows: If c be the distance of k from the centroid of the pendulum then

$$a^2(1 + R) = \alpha^2 + (c - b)^2 + R\beta^2 + R(c - d)^2.$$

Also, since

$$R = \frac{(c - b)}{(d - c)},$$

then

$$(d - c) = \frac{(d - b)}{(1 + R)} \quad \text{and} \quad (c - b) = \frac{R(d - b)}{(1 + R)}.$$

Substituting these values in the above expression the value of a^2 is found to be

$$a^2 = \frac{(\alpha^2 + \beta^2 R)}{(R + 1)} + \frac{R(b - d)^2}{(R + 1)^2}, \tag{44}$$

[1] In Part I., PHYS. REV., Vol. XXV., pp. 274–293, the general equations of the pendulum were derived and discussed under three divisions. These equations are as follows:

$$\frac{gt^2}{\pi^2}(b + Rd) = Rd^2 + b^2 + \alpha^2 + R\beta^2, \tag{10}$$

$$\frac{gt^2}{\pi^2}[L(1 + R) - (b + Rd)] = R(L - d)^2 + (L - b)^2 + \alpha^2 + R\beta^2. \tag{11}$$

By substituting this value of a^2 and the value of z' given in equation (9) into the expression $z'z'' = a^2$ the value of z'' is found to be:

$$z_1'' = \frac{(\alpha^2 + R\beta^2)}{(b + Rd)} + \frac{R(b - d)^2}{(R + 1)(b + Rd)}, \tag{45}$$

an expression in which z'' and d are the only variables.

Equation (9) appears graphically as a straight line; equation (45) as a hyperbola with a vertical asymptote a distance b/R to the left of the Y axis.

If now a new axis of suspension be chosen, at a distance L, a new set of four parallel axes having a common period will be determined, and the corresponding z values will be given by the equations

$$z_2' = \frac{(L - b) + R(L - d)}{R + 1}, \tag{46}$$

$$z_2'' = \frac{\alpha^2 + R\beta^2}{(L - b) + R(L - d)} + \frac{R}{R + 1} \cdot \frac{(b - d)^2}{(L - b) + R(L - d)}. \tag{47}$$

z_2' and z_2'' are here taken positive when the centroid is to the left of k_2. The graphs of equations (9), (45), (46) and (47) are shown in Fig. 9, for a pendulum whose constants are as follows:

$$R = 1, \quad L = 60, \quad b = 40, \quad \alpha^2 + R\beta^2 = 600.$$

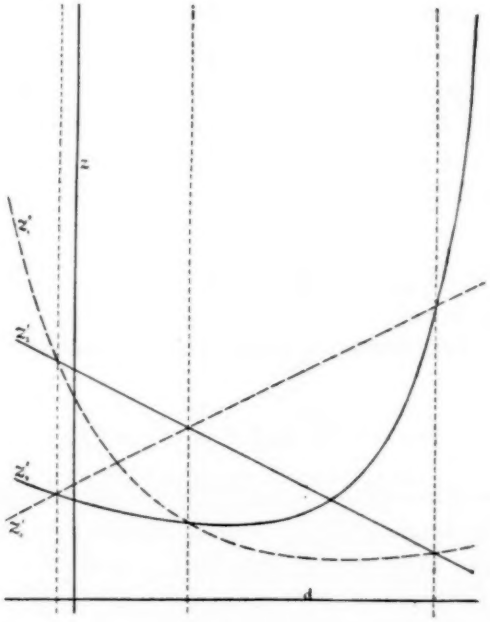

Fig. 9.

Of the ten intersections of the four graphs—two of the intersections being outside of the limits of the figure—six are paired for three values of d, and each of the six is between a z_1 and a z_2 curve. This means that whenever one of the four k_1 axes coincides with a k_2 axis, each of the three other k_1 axes coincides with one of the three remaining k_2 axes and thus the period is the same for all. Thus for the three values of d, the periods from k_1 and k_2 are equal. Further, for any value of d, the sum of the ordinates of the two straight lines is equal to L, as appears from equations (9) and (45); therefore the sum of the ordinates of the points of intersections corresponding to the first and last values of d is likewise equal to L, or $z_1' + z_1'' = z_2' + z_2'' = L$, and the pendulum is in Kater adjustment. From similar considerations it may be shown that this cannot, in general, be true of the middle value of d. It is true however if the two points of intersection coincide, and consequently,

$$z_1' = z_1'' = z_2' = z_2'' = a.$$

Thus there are in general three values of d for which the pendulum swings with equal periods from k_1 and k_2, for two of which it is in Kater adjustment, and for one of which, in general, it is not. This last is the condition designated as the "third intersection" in the discussion of equations (10) and (11) when t and d are taken as the variables.

DIVISION I., t AND d TAKEN AS THE VARIABLES.

Equations (10) and (11), as previously shown, are cubics belonging to the 53d species named in Newton's classification. The general form is shown by the full line curve in Fig. 10A, where values of d are plotted as abscissæ and t as ordinates. As t must be positive, only that portion of the graph which lies above the X axis is realizable. Rewriting equation (10) in the form

$$t^2 = \frac{\pi^2}{g}\left(d - \frac{b}{R}\right) + \frac{b^2/R + b^2 + \alpha^2 + R\beta^2}{b + Rd}$$

it is seen that for large values of d the curve indefinitely approximates the parabola

$$\frac{gt^2}{\pi^2} = d - \frac{b}{R}.$$

This parabola is shown in the dotted curve of Fig. 10A, where its asymptotic character is clearly shown. Also, as d approaches the value $-b/R$, t increases without limit in both directions, giving

$$d + \frac{b}{R} = 0$$

as a vertical asymptote. It will be convenient to refer to the *vertical* and *parabolic* arms of this curve, meaning thereby those parts having the vertical and parabolic asymptotes respectively.

As already shown three only of the nine intersections of equations

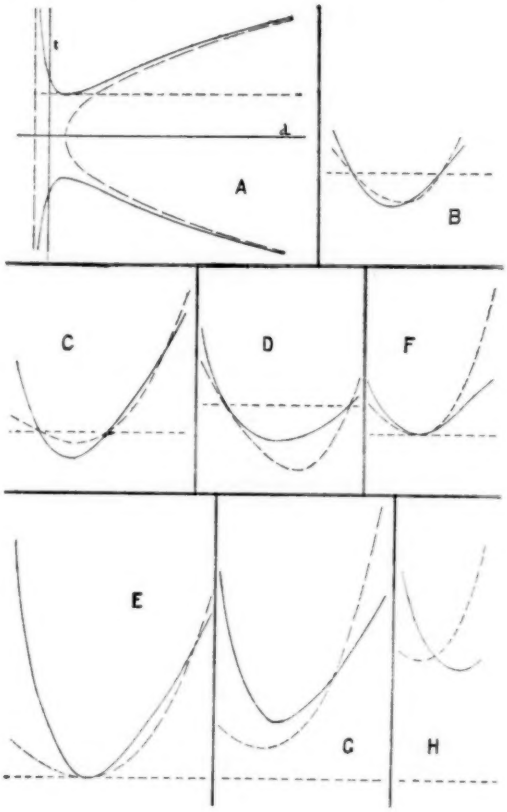

Fig. 10.

(10) and (11) belong to the present discussion. Their coördinates are given by equation (27). Two of the three intersections fall on the line

$$t = \pi \sqrt{\frac{L}{g}}$$

(the dotted horizontal line in the graphs of Fig. 10), and indicate Kater adjustment: while the third always falls on the line

$$d = \frac{L}{2} + \frac{(L - 2b)}{2R}.$$

The first two are disposed symmetrically about the line $d = L/2$. While these are sometimes imaginary, the third intersection is always real. It

may be on either side of, between, or coincident with the other two. These conditions are illustrated in Fig. 10, curves B to H.

As a check on this part of the theory an experiment was performed with a reversible pendulum arranged to give $d_3 > d_1$. The constants of the pendulum had the following values:

$$\alpha^2 = 610.4, \quad \beta^2 = 3.14, \quad R = 0.43, \quad b = 10.4, \quad L = 50.$$

Introducing these constants into equations (10) and (11) the following working equations are had:

$$\left.\begin{aligned} t_1^2 &= \frac{0.01006(d^2 + 1673.7)}{(d + 24.178)}, \\ t_2^2 &= \frac{0.01006[(50 - d)^2 + 5067.9]}{(142.063 - d)}. \end{aligned}\right\} \tag{48}$$

The results obtained both by experiment and calculation are given in the following table.

d	t_1 Experimental.	t_1 Calculated.	t_2 Experimental.	t_2 Calculated.
5	0.767	0.765	0.719	0.722
10	0.720	0.723	0.712	0.713
15	0.697	0.698	0.705	0.706
20	0.687	0.687	0.701	0.701
25	0.685	0.686	0.698	0.699
30	0.690	0.691	0.700	0.701
35	0.702	0.702	0.704	0.705
40	0.716	0.716	0.714	0.714
45	0.733	0.733	0.726	0.727
54	0.769	0.769	0.763	0.762
57	0.782	0.781	0.778	0.778
60	0.794	0.794	0.797	0.796
65	0.816	0.816	0.833	0.831

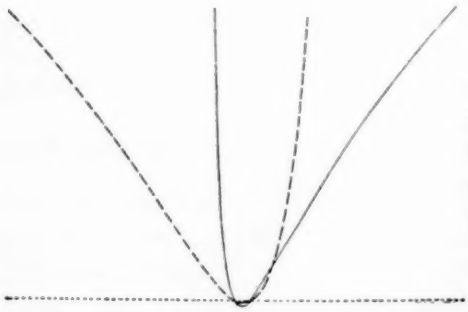

Fig. 11.

Fig. 10c is drawn from these data.

By the use of equations (48) these curves may readily be exterpolated to show the characteristics of the typical curve A in Fig. 10. This is done in Fig. 11, where d is extended to $\pm$ 330. For purposes of comparison the ratio between the scales of ordinates and abscissæ is kept the same in Fig. 11 as in Fig. 10c.

In discussing the possible arrangements of the three intersections of equations (10) and (11) it is convenient to consider their minimum points as the constants of the pendulum are varied. The coördinates (designated as δ_1, τ_1 and δ_2, τ_2) are as follows:

$$\left.\begin{aligned} \delta_1 &= -\frac{b}{R} + \left[\frac{b^2}{R^2} + \frac{b^2+\alpha^2}{R} + \beta^2\right]^{\frac{1}{2}}, \\ \tau_1 &= \pi\sqrt{\frac{2}{g}}\left\{-\frac{b}{R} + \left[\frac{b^2}{R^2} + \frac{b^2+\alpha^2}{R} + \beta^2\right]^{\frac{1}{2}}\right\}^{\frac{1}{2}}, \\ \delta_2 &= L + \frac{L-b}{R} - \left[\frac{(L-b)^2}{R^2} + \frac{(L-b)^2+\alpha^2}{R} + \beta^2\right]^{\frac{1}{2}}, \\ \tau_2 &= \pi\sqrt{\frac{2}{g}}\left\{-\frac{L-b}{R} + \left[\frac{(L-b)^2}{R^2} + \frac{(L-b)^2+\alpha^2}{R} + \beta^2\right]^{\frac{1}{2}}\right\}^{\frac{1}{2}}. \end{aligned}\right\} \quad (49)$$

From these equations the loci of the minimum points may be derived, and are as follows:

$$\frac{g\tau_1^2}{\pi^2} = 2\delta_1, \quad (50)$$

$$\frac{g\tau_2^2}{\pi^2} = 2(L - \delta_2). \quad (51)$$

It will at once be noticed that the first of these equations is independent of all of the constants of the pendulum while the second one is dependent only upon L, and in such a way as merely to displace the curve along the axis of abscissas. Both curves are parabolæ, are of the same size, but are turned in opposite directions. The vertex of the first one is at (0, 0), while that of the second one is at $(L, 0)$: the X-axis is the axis of both curves. The somewhat remarkable conclusion is reached that every reversible pendulum, no matter how constructed, in which t and d are taken as variables, has the minimum-points of its period-curves upon two parabolæ of fixed size, the distance apart of the vertices being the only variable quantity.

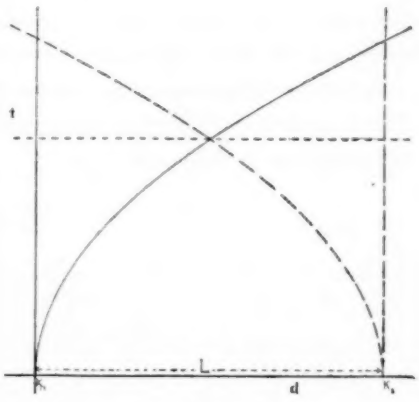

Fig. 12.

A study of the graph of these curves (Fig. 12) leads at once to several interesting conclusions. Thus the curves always intersect at the point $(L/2, \pi\sqrt{L/g})$, *i. e.*, on the g-line and halfway between the knife-edges:

again, when the minimum points are above the g-line the k_1 minimum is to the right of the k_2 minimum; when they are below the reverse is the case. Also both minimum points must be either above or both below the g-line. This follows from the fact that the graphs of equations (10) and (11) cross the g-line at the same two points (Kater intersections), and if these intersections are real for one curve they must be for the other also. It may also be shown, by introducing the condition $\tau_1 = \tau_2$ into equations (50) and (51), that the ordinates of the minimum-points cannot be equal unless either the centroid of the fixed part is midway between the knife-edges, or the two minimum points coincide on the g-line. An interesting theorem may be noted concerning the relative positions of the minimum-points and the third intersection as follows: If the abscissa d_3 of the third intersection is to lie between those of the minimum points, *i. e.*, $\delta_1 \gtrless d_3 \gtrless \delta_2$ then the quantity $L/2 \cdot (R + 1)/R$ must be either greater or less than both of the quantities $[b^2/R^2 + (b^2+\alpha^2)/R + \beta^2]^{\frac{1}{2}}$ and $[(L - b)^2/R^2 + [(L - b)^2 + \alpha^2]/R + \beta^2]^{\frac{1}{2}}$ according as the minimum points are below or above the g-line. For the abscissa of the third intersection to lie to the right or the left of the minimum points, *i. e.*,

$$d_3 \gtrless \begin{cases} \delta_1 \\ \delta_2 \end{cases}$$

the quantity $L/2 \cdot (R + 1)/R$ must be intermediate in value between the above two radicals.

It is now possible to state the relations necessary between the arbitrary constants of the pendulum, in order to produce the conditions illustrated by graphs B–H in Fig. 10.

Curve B.—Third intersection between the first and second: for this it is sufficient that the third intersection be below the g-line, or that $t_3 < \pi\sqrt{L/g}$. This reduces to the condition

$$L^2 > (L - b)^2 \cdot R + \frac{4(\alpha^2 + R\beta^2)}{(R + 1)}. \tag{52}$$

Curve C.—The third intersection to the right or left of both of the others, *i. e.*, $d_3 > d_1$, or $d_3 < d_2$ (d_1 is always greater than d_2).

One of these conditions will always hold when,

$$L^2 > \frac{4b(L - b)}{1 - R} - \frac{4R(\alpha^2 + R\beta^2)}{1 - R^2}. \tag{53}$$

The third intersection will be to the right when in equation (53) $L/2 > b$, and it will be to the left when $L/2 < b$; for, since the third intersection requires that the centroid of the whole pendulum be midway between the knife-edges, it is evident that the centroids of the fixed and movable parts must be on opposite sides of this mid-way point.

Curve D.—The third intersection coincident with either the first or the second intersection. In this case,

$$b = \frac{L}{2} \pm \frac{1}{2}\left[\frac{RL^2(R+1) - 4R(\alpha^2 + R\beta^2)}{R+1}\right]^{\frac{1}{2}}, \tag{54}$$

the negative sign applying when $d_3 = d_1$, and the positive sign when $d_3 = d_2$.

Curve E.—The first and second intersections to coincide with each other but not with the third: or

$$d_1 = d_2 \quad \text{and} \quad \frac{L}{2} \lesseqgtr d_3.$$

This requires $b \neq L$, and

$$b = \frac{L}{2} \pm \left[\frac{L^2}{4}(R+1) - (\alpha^2 + R\beta^2)\right]^{\frac{1}{2}}. \tag{55}$$

For $d_3 > L/2$ the positive sign in (55) applies, and for $d_3 < L/2$ the negative sign.

Curve F.—All three intersections to coincide. In this case

$$d_1 = d_2 = d_3 = \frac{L}{2}.$$

This requires $b = L/2$, and condition (55), which gives

$$\frac{L}{2} = \left[\frac{\alpha^2 + R\beta^2}{R+1}\right]^{\frac{1}{2}}. \tag{56}$$

Curve G.—The first and second intersections imaginary and the third to be between a vertical and a parabolic arm. The necessary condition is

$$\left[\frac{b^2}{R^2} + \frac{b^2+\alpha^2}{R} + \beta^2\right]^{\frac{1}{2}} \lesseqgtr \frac{L}{2}\cdot\frac{R+1}{R} \lesseqgtr \left[\frac{(L-b)^2}{R^2} + \frac{(L-b)^2+\alpha^2}{R} + \beta^2\right]^{\frac{1}{2}}, \tag{57}$$

the upper signs holding when the intersection falls to the right and the lower when it falls to the left.

Curve H.—The first and second intersections imaginary, and the third to be between vertical arms. The necessary condition is

$$\left.\begin{aligned} \frac{L}{2}\cdot\frac{R+1}{R} &< \left[\frac{b^2}{R^2} + \frac{b^2+\alpha^2}{R} + \beta^2\right]^{\frac{1}{2}}, \\ \frac{L}{2}\cdot\frac{R+1}{R} &< \left[\frac{(L-b)^2}{R^2} + \frac{(L-b)^2+\alpha^2}{R} + \beta^2\right]^{\frac{1}{2}}. \end{aligned}\right\} \tag{58}$$

If $b = L/2$, then $d_3 = L/2$, or the movable mass will be midway between the knife-edges when the third intersection is obtained, while $b > L/2$ throws it to the left and $b < L/2$ to the right.

The constants of the pendulum used in computing the graphs of Fig. 10 were as follows:

Curve.	α^2	β^2	R	b	L
B	610.4	3.14	0.430	21	45
C	610.4	3.14	0.430	10.4	50
D	610.4	3.14	0.430	33.3	50
E	610.4	3.14	0.430	8.2	50
F	610.4	3.14	0.430	20.68	41.35
G	610.4	3.14	0.430	10.4	40
H	610.4	3.14	0.430	16.0	30

A study of this table brings out several interesting points in connection with the curves of Fig. 10. Thus it will be noticed that curves C, D, and E are produced with the same constants except for variations in b: while curves C and G differ only in L, so that the k_1 curve is the same in both while the k_2 curve is displaced along the Y-axis.

A laboratory experiment often performed with the compound pendulum is to vary the position of the axis of suspension leaving all else constant. This is accomplished by suspending the pendulum bar on a fixed knife-edge passing through one of a series of holes cut in the bar to receive it, thus changing the axis without altering the distribution of mass (vide Nichols' or Duff's laboratory manuals). Thus L and b (or L and $L - b$) are varied and with them the period. As there is no movable mass d, R and β become zero. The variables are t and b and the case may be considered as a special one under the present discussion. It will be noticed that L occurs in equation (11) only in the expressions $L - b$ and $L - d$. This difference remains invariable as k_1 is moved, giving no variation in the period from the fixed knife-edge. Equation (10) may then be rewritten in the form

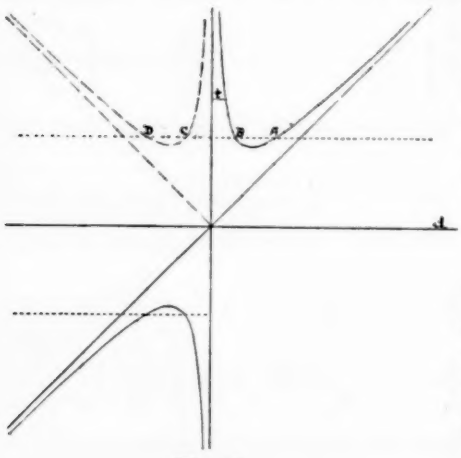

Fig. 13.

$$\frac{gt^2}{\pi^2} = \frac{\alpha^2 + b^2}{b} = l, \qquad (58)$$

where l is the equivalent length of the pendulum. The application of the graphical method to this case while rather simple, brings out several interesting points. Let values of b be platted as abscissas and values of l as ordinates. A hyperbola results, as shown in Fig. 13, having as

asymptotes the lines $b = 0$ and $b = l$. As the pendulum reverses when the axis of suspension passes over the centroid, taken as the origin in the graph, it is convenient to "turn up" the third quadrant into the second, as shown by the dotted lines on the curve. It will now be seen that a minimum occurs when $b = \alpha$, and in that case $l = 2\alpha$, showing that the minimum period for any pendulum is given by the condition

$$t = \pi \sqrt{\frac{2\alpha}{g}}. \tag{59}$$

The graph for the fixed axis of suspension is given by the two straight lines $l = \pm l'$, the sign depending upon which side of the centroid the fixed axis is taken. Further if $l' > 2\alpha$, there are four intersections between the straight lines and the hyperbola, indicating four positions of the movable axis, A, B, C, D, which give the same equivalent length, l'. One of the points will coincide with the position of the fixed axis. As already shown, A and C, and also B and D, are l' cm. apart, and correspond to the Kater adjustment. If $l' < 2\alpha$ then the line $l = l'$ is tangent to the hyperbola and the four positions A, B, C, D reduce to two, the minimum points, 2α cm. apart. If $l' < 2\alpha$ there are no intersections and Kater adjustment is impossible.

Division II., t and β Taken as Variables.

As a method of determining the value of g this method has advantages as it is possible by taking four periods to determine g immediately. It is not necessary that the pendulum be brought into Kater adjustment, or even capable of such adjustment.

In this method the movable mass is made up of two masses m, m, which are slid along the pendulum rod as shown in Fig. 5, in such a way as always to be equidistant from a given point, d cm. from k_1. Thus β may be varied and consequently t, without the variation of any other of the quantities entering into equations (10) or (11). In the graphical treatment, using equations (30) and (33), t^2 is plotted as y and q^2 as x, $2q$ being the distance between the movable masses m, m. As shown in Fig. 6 the graphs reduce to two straight lines, which intersect on the line

$$y = t^2 = \pi^2 \frac{L}{g},$$

and this intersection point may be used in the determination of g. As the moment of inertia of the pendulum is a minimum when $q = 0$, in this condition the equivalent length must be no greater than L for Kater adjustment to be possible. This leads to the condition,

$$L > [(\beta_0^2 + d^2)R + b^2 + \alpha^2] \div (b + Rd). \tag{60}$$

It is interesting to note that with this form of pendulum the condition signified by the third intersection, in Division I., is possible only when the centroid of the whole pendulum has first been adjusted midway between the knife-edges. If this be done the intersection secured will be the "third" and the Kater adjustment will be impossible, unless the pendulum is in the condition indicated by the coincidence of all three intersections as discussed in Division I. This will occur when β for the point of intersection has the value

$$\beta^2 = \frac{1}{R}\left[\left(\frac{(R+1)L^2}{4} - \alpha^2\right) - \left(b - \frac{L}{2}\right)^2 - R\left(d - \frac{L}{2}\right)^2\right]. \quad (61)$$

If $b = L/2$, then $d = L/2$, and this reduces to condition F, Division I., *i. e.*,

$$\beta^2 = \frac{1}{4R}[(R+1)L^2 - 4\alpha^2].$$

The following experiment was performed as a check on the theory: A pendulum was arranged with the following constants:

$$L = 60, \quad R = 1.51, \quad d = 0, \quad b = 15, \quad \beta_0^2 = 3.14, \quad \alpha^2 = 628.5.$$

β_0^2 was computed from the dimensions and mass of m, and α^2 from determinations of the periods from the two knife-edges, employing the fixed part alone, and using the equations,

$$\alpha^2 = b\left(\frac{gt^2}{\pi^2} - b\right) = (L-b)\left[\frac{gt^2}{\pi^2} - (L-b)\right].$$

As these constants fulfil the conditions of equation (60) the intersection will fall in the first quadrant.

By substituting the above constants into equations (30) and (33) the following working equations are obtained:

$$\left.\begin{aligned} t_1^2 &= 0.001013q^2 + 0.5759, \\ t_2^2 &= 0.000112q^2 + 0.6005. \end{aligned}\right\} \quad (62)$$

The following data were secured, and the graph is shown in Fig. 14*a*.

q^2	Experimental Values.		Calculated from Equation (62).	
	t_1^2	t_2^2	t_1^2	t_2^2
25	0.6005	0.6037	0.6012	0.6033
400	0.9811	0.6472	0.9811	0.6453

Determining analytically the value of t_2 for the intersection gives $t_2 = 0.60403''$ and solving for g gives the value $g = 980.61$.

In a second experiment to illustrate the case of exterpolation, when the Kater adjustment becomes impossible the following constants were used:

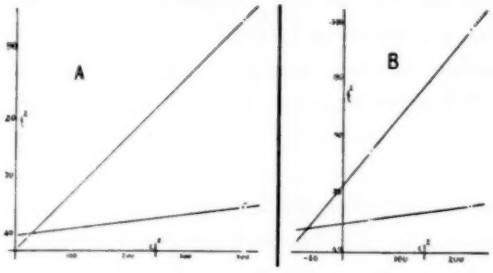

Fig. 14.

$L = 64, \quad R = 2.28, \quad d = -2.5, \quad b = 25, \quad \beta_0^2 = 3.14, \quad \alpha^2 = 743.9.$

The working equations are

$$\left.\begin{aligned} t_1^2 &= 0.00119q^2 + 0.7252, \\ t_2^2 &= 0.0001204q^2 + 0.65175. \end{aligned}\right\} \tag{63}$$

The graphs are shown in Fig. 14*b* and the experimental and calculated values below:

q^2	Experimental Values.		Calculated from Equation (63).	
	t_1^2	t_2^2	t_1^2	t_2^2
49	0.7779	0.6585	0.7835	0.6577
225	0.9940	0.6831	0 9930	0.6788

Determining g by substitution as above gives $g = 982.3$, a result well within the limits of accuracy, as the experiment was conducted.

Division III., t and R Taken as Variables.

In the third division of the discussion of equations (10) and (11) it was found necessary to modify the pendulum so as to make the value of β^2 very large in certain cases. Fig. 15 shows the form finally used. The

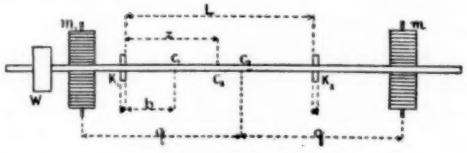

Fig. 15.

masses m and m, constituting the variable mass, are made up of brass discs which may be threaded onto appropriate supports. Discs of two sizes were used, one 7.67 cm. and the other 10.17 cm. in diameter. By

varying the number of discs, the value of R may be varied without changing any other of the constants of the pendulum. One of the disc supports remained fixed at one end of the pendulum while the other could be moved to any desired point, or left off altogether. When both were in use, an equal number of discs was placed on each, thus keeping the centroid of the variable mass half way between the supports.

If β_0 be the radius of gyration of one of the cylinders about its own axis, and $2q$ the distance between cylinders then the square of the radius of gyration of the variable mass is $\beta^2 = \beta_0^2 + q^2$. If but one disc support be in use then $\beta^2 = \beta_0^2$. In the pendulum employed β_0^2 could be given three values, in the neighborhood of 10, the value of q could be changed through a wide range, its maximum value being about 44 cm., giving $\beta^2 = 1{,}940$ ±. A chronograph was available for recording the periods.

The results of four typical experiments are given, illustrating the four curves of Fig. 7. On page 291, Vol. XXV., a table is given showing the combinations of the constants of the pendulum necessary to produce the four cases. The following table gives the values selected in order to fulfil these conditions:

	L	b	d	a^2	β^2
I.	60	50.75	13.25	961.90	7.403
II.	60	17.75	24.85	600.22	1,933.8
III.	60	27.35	20.00	829.00	7.403
IV.	56	56.30	14.00	829.00	11.57

Below are given the data obtained, and in parenthesis immediately after, the corresponding value computed by means of the accompanying theoretical equations.

I.

$$t_1^2 = \frac{0.138895(R + 19.3462)}{(R + 3.8302)},$$

$$t_2^2 = \frac{0.4721(R + 0.4773)}{(R + 0.19786)}.$$

R	t_1		t_2	
0.0	0.8376	(0.8374)	1.0677	(1.0673)
0.067	0.8317	(0.8316)	0.986	(0.984)
0.135	0.8262	(0.8258)	0.932	(0.935)
0.202	0.8205	(0.8203)	0.8955	(0.8956)
0.268	0.8154	(0.8151)	0.8686	(0.8690)
0.335	0.8103	(0.8106)	0.8480	(0.8480)
0.402	0.8051	(0.8048)	0.8316	(0.8319)
0.468	0.8012	(0.7999)	0.8185	(0.8186)
0.533	0.7954	(0.7954)	0.8063	(0.8077)
0.598	0.7910	(0.7907)	0.7986	(0.7974)

R	t_1		t_2	
0.663	0.7865	(0.7863)	0.7906	(0.7904)
0.729	0.7824	(0.7812)	0.7838	(0.7838)
0.793	0.7780	(0.7778)	0.7780	(0.7779)
0.861	0.7739	(0.7735)	0.7728	(0.7725)
0.927	0.7698	(0.7694)	0.7681	(0.7677)
0.993	0.7659	(0.7653)	0.7639	(0.7634)

II.

$$t_1^2 = \frac{1.03285(R + 0.35819)}{(R + 0.71268)},$$

$$t_2^2 = \frac{0.907066(R + 0.75368)}{(R + 1.20313)}.$$

R	t_1		t_2	
0.0	0.7205	(0.7205)	0.7538	(0.7538)
0.358	0.8309	(0.8310)	0.8041	(0.8037)
0.716	0.8802	(0.8812)	0.8344	(0.8335)
1.075	0.9102	(0.9100)	0.8541	(0.8533)
1.433	0.9292	(0.9286)	0.8686	(0.8674)
1.792	0.9432	(0.9417)	0.8793	(0.8781)
2.151	0.9527	(0.9513)	0.8876	(0.8863)
2.509	0.9609	(0.9588)	0.8941	(0.8929)
2.867	0.9671	(0.9647)	0.8997	(0.8983)
3.224	0.9724	(0.9695)	0.9042	(0.9017)
3.580	0.9767	(0.9734)	0.9072	(0.9066)

III.

$$t_1^2 = \frac{0.204966(3.8722 + R)}{(1.368 - R)},$$

$$t_2^2 = \frac{0.80611(0.2958 + R)}{(0.408 + R)}.$$

R	t_1		t_2	
0.0	0.7618	(0.7617)	0.7644	(0.7644)
0.065	0.7870	(0.7870)	0.7843	(0.7841)
0.130	0.8128	(0.8140)	0.7965	(0.7988)
0.196	0.8433	(0.8436)	0.8105	(0.8102)
0.261	0.8748	(0.8742)	0.8199	(0.8191)
0.326	0.9091	(0.9087)	0.8285	(0.8264)
0.392	0.9464	(0.9463)	0.8333	(0.8325)
0.458	0.9876	(0.9875)	0.8386	(0.8377)
0.524	1.0337	(1.0333)	0.8431	(0.8421)
0.591	1.0852	(1.0851)	0.8469	(0.8459)
0.658	1.1439	(1.1436)	0.8503	(0.8493)
0.724	1.2106	(1.2095)	0.8531	(0.8522)
0.792	1.2884	(1.2883)	0.8560	(0.8548)
0.859	1.3806	(1.3803)	0.8586	(0.8572)
0.926	1.4929	(1.4917)	0.8606	(0.8593)

IV.

$$t_1^2 = \frac{0.14899(R + 19.579)}{(R + 4.0214)},$$

$$t_2^2 = \frac{0.42524(R + 0.5008)}{(R - 0.007)}.$$

R	t_1		t_2	
0.0	0.8518	(0.8517)	c above k	
0.262	0.8307	(0.8307)	1.1259	(1.1278)
0.525	0.8117	(0.8117)	0.9173	(0.9176)
0.788	0.7943	(0.7943)	0.8379	(0.8377)
1.05	0.7785	(0.7785)	0.7972	(0.7951)
1.31	0.7640	(0.7640)	0.7690	(0.7687)
1.57	0.7507	(0.7508)	0.7510	(0.7503)
1.83	0.7384	(0.7383)	0.7367	(0.7370)
2.09	0.7268	(0.7268)	0.7282	(0.7270)
2.35	0.7157	(0.7161)	0.7197	(0.7190)

The condition indicated by the "third" intersection in Division I. is realizable in Division III. whenever it is possible by varying R, to throw the centroid of the whole pendulum midway between the knife-edges. This value of R, from equation (9), is given by the expression

$$R = -\frac{(L - 2b)}{(L - 2a)}. \qquad (64)$$

To make R positive in this equation, it is further necessary that $b > L/2 > d$, or that $b < L/2 < d$. The verification of theory by experiment in the above tables is all that could be asked for. Figure 16 shows the graphs corresponding to the above four cases.

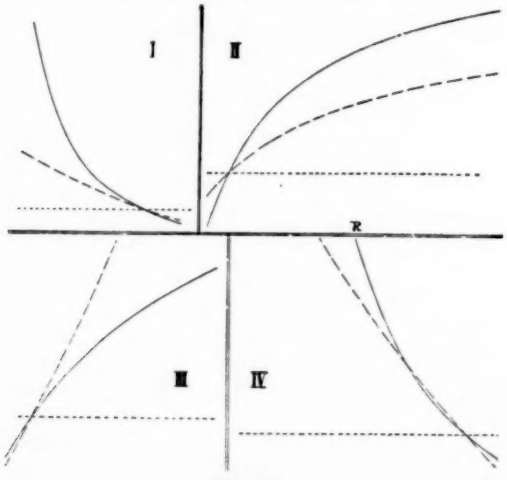

Fig. 16.

In a third paper it is proposed to discuss Best Values and to give a short critique of Kater's classic experiments.

September 1, 1911.

PROCEEDINGS

OF THE

AMERICAN PHYSICAL SOCIETY.

THE PRIMARY CONCEPTS OF PHYSICS.[1]

BY WILLIAM FRANCIS MAGIE.

THE subject of the present address is one that does not often appear on a scientific programme. Physicists are so busy in enlarging the structure of knowledge that few of them concern themselves with the consideration of the fundamental concepts of the science. Yet it is plainly true that if those fundamental or primary concepts are not clearly apprehended, or if there is doubt as to what they are, the whole structure of the science rests on an insecure basis. I propose to examine certain questions concerning these primary concepts, about which there has been and is much unsettled opinion. The discussion necessarily rests upon my own beliefs about them. In the nature of the case each man can speak positively about them for himself only. It would be very improper to dogmatize, and I shall accordingly have to crave your pardon for a frequent expression of my own opinion, believing it less objectionable to be egotistic than to be dogmatic.

The first question which I shall consider is that raised by the advocates of the dynamical definition of force, as to the order in which the concepts of force and mass come in thought when one is constructing the science of mechanics, or in other words, whether force or mass is the primary concept. It will be of service in the discussion if we consider briefly the way in which some of the great builders of the science of mechanics used these concepts.

There is no need of presenting the views of Archimedes or of Stevinus, whose work was exclusively in statics and who used the concept of force given us by our muscular or motor sense, and measured forces by weights. The views of Galileo, however, are interesting as showing how far one can go in dynamics without using the concept of mass.

Galileo examined the problem of the motion of a body acted on by a constant force. The only constant force of which he could dispose was the weight of a body, or a component of its weight, and he accordingly was limited in his studies to the examination of the laws of falling bodies. Owing to the relation of proportionality between the weight of a body

[1] Presidential address delivered before the American Physical Society at Washington, D. C., December 28, 1911.

and its mass, this limitation in a way simplified the problem, while at the same time it made it more difficult to develop a complete doctrine of force and motion. By the famous experiment at the Leaning Tower Galileo satisfied himself that he could study any falling body as a type, and that the conclusions which he would reach from that study would apply to all. His attention was therefore directed almost wholly to the consideration of the motion of the falling body, while the question of the relation between the motion and the weight of the body was disregarded. The result of this was that he developed the laws of linear motion with constant acceleration, and numerous consequences of those laws, chiefly relating to motion down inclined planes, with really wonderful completeness, and was led in the course of his thought to a full appreciation and statement of the principle of inertia, while yet he did not, in this part of his work, attain to any useful conception of the relation of force to mass. He makes it clear that the conception of force which is sufficient for his purposes is that with which he was familiar from his study of statics. He says, in speaking about the "tendency" of a body to fall down inclined planes of the same height, that "It is clear that the tendency of a body to fall is as great as the resistance or the least force which suffices to prevent its falling and to keep the body at rest." In fact Galileo thought of the weight of a body, with which he was familiar from common experience, as a force which moved the body, and assuming that the weight was unchanged during motion his experiments demonstrated what kind of motion such a constant force will set up and maintain.

In the very interesting discussion which Galileo gives of the forces exerted by the collision of one body against another, he approaches nearer than in other parts of his discourse to an appreciation of mass as a characteristic of a moving body. He speaks in one place of the falling body being a composite of weight and velocity, and his discussion of the impulse applied by such a falling body to another on which it falls shows that he was very near the concept of momentum; but there is no real precision in his statements.

We now turn to Newton to get the full doctrine of the relations of force and motion. It will be clear to anyone who examines the introductory parts of the Principia, that Newton did not undertake in that book to present a systematic treatise on dynamics. He merely blocks out a rough set of definitions and postulates, in a very uncritical way, which are sufficient to enable him to go on as promptly as possible to the real task which was before him. A striking instance of this uncritical attitude of mind is found in Definition I., in which he says "Quantity of matter is its measure derived from its density and volume jointly." This quantity of matter thus defined he names mass. Since we can only define density in terms of the concept of mass it is surely uncritical to define mass in terms of density. In fact Newton on a later page uses the true definition when he says that bodies are of the same density, if their *vires inertiæ* (that is, their masses) are proportional to their volumes.

The same sort of uncritical treatment appears in his presentation

of the various types of force. He mentions first the *vis insita*, which he defines as the power of resisting, by which a body persists in its state of rest or of uniform motion. He says it differs in no respect, except in the way of conceiving of it, from the inertia of a mass.

Then comes *vis impressa*, the action (*actio*) exerted on a body to change its state of rest or of uniform motion. This is force in our ordinary sense. Newton says that it arises from a blow, from pressures or from centripetal force.

Vis centripeta is the force by which bodies are drawn or impelled from all directions towards any point as a center, or tend toward it in any way whatever. The force of gravitation and magnetic force are examples of this centripetal force. So also is the force by which a sling draws a stone in it toward the hand, which force Newton explains as arising from the stretching of the cord of the sling.

Newton then goes on to define the various measures or modes of giving quantitative expressions for centripetal forces. He first describes the *vis centripetæ quantitas absoluta* as the measure of it as greater or less by comparison with the efficiency of the cause which transmits it from the center through the surrounding region. Thus the magnetic force is greater in one magnet than in another, either because of the greater mass of the one, or of the intensity of its power.

The *vis centripetæ quantitas acceleratix* is the measure of it as proportional to the velocity, which it generates in a given time. Thus the power (*virtus*) of a magnet is greater at lesser distances, and less at greater distances; gravitating force is greater in valleys, less on mountain peaks, and less still at greater distances from the earth. At equal distances, he says, this gravitating *vis acceleratrix* is the same everywhere, because all falling bodies are equally accelerated.

The *vis centripetæ quantitas motrix* is the measure of it as proportional to the momentum which it generates in a given time. This quantity is the center-seeking or tendency to the center of the whole body, and (as Newton says, with an evident appreciation that he is limiting the generality of his conception) is the weight of the body. It is always known by the force opposite to it, and equal to it, by which the fall of the body can be prevented.

Newton calls these quantities of force of the various sorts described by the shorter terms motive, accelerative, and absolute forces, that is, he substitutes the general terms for the measured quantities of the forces which can be conceived only in those general terms. With this understanding he states that the *vis acceleratrix* is to the *vis motrix* as velocity is to momentum; for the quantity of motion (*momentum*) arises from the velocity and quantity of matter, and the *vis motrix* arises from the *vis acceleratrix* and the quantity of matter. For the sum of the actions of the *vis acceleratrix* upon the several particles of a body is the *vis motrix* of the whole body. Newton relates the *vis motrix* to a body as a striving of the whole body towards the center, made up of the strivings of all its parts; the *vis acceleratrix* to the position of the body, as a certain efficiency, diffused from the center through all places

around it, for moving bodies which are in those places. The *vis acceleratrix* as thus described reminds us of the conception of the strength of a field of force.

This analysis of the concept of force surely does not promote a clear apprehension of it. The only one of the distinctions which have been made which seems to be worth retaining is that between the *vis impressa* or action, and the *vis motrix impressa*, the one being force in its general or conceptual sense, the other the same force when given a measure or value. This distinction was clearly in Newton's mind and appears in the enunciation of the Laws of Motion. In the First Law the departure of a body from its state of rest or of uniform motion is ascribed to the *vis impressa;* that is, to force in general, without any specification as to its measure or even any declaration that it can be measured. In the Second Law the change of momentum is said to be proportional to the *vis motrix impressa;* that is, to force that is measured so that a proportionality to something else can be predicated of it. As has already been stated, Newton declared of this *vis motrix* in the special case of gravitation that it is known, or measured, by the force opposite to it and equal to it, by which the fall of the body, or, in the general case, the motion of the body, can be prevented. In the Third Law the force is called *actio.* This is the alternative word used in the definition of the *vis impressa*, as an equivalent for force in its general sense. The word in this sense is consistently used in the enunciation of the Third Law, in which forces are not considered as measured, but merely as compared by the condition of equality. From the examples of action and reaction which Newton gives (the finger pressed against a stone; the horse drawing a stone by a rope and drawn back equally toward the stone, because of the stretching of the rope and its exertion of equal forces at its two ends) it is plain that Newton conceived of forces in the way which is familiar to all of us, as the pushes and pulls which can be perceived by our motor sense, and as the causes of motions. He goes on to say that by these *actions* there are caused equal changes, not of velocity, but of momentum, so that the changes of velocity are inversely as the bodies (*corporibus*). In this way, without measuring forces, there is introduced the method of comparing masses.

It is difficult to perceive in these many definitions and declarations exactly what Newton's conception was of force, of the unit in which it is measured and of its relation to mass. After careful consideration of all that I can find in the Principia bearing on the question I am convinced that Newton viewed the concept of force as a primary one, or one directly given by intuition, and that he thought of the motions of bodies caused by these forces as connected quantitatively with them by the experimental relation embodied in the Second Law. Since Newton does not use a system of units, and states most of his laws and theorems in terms of proportions, the priority of force to mass, in the order of their apprehension, is not clearly presented.

In the matter of measuring a force he clearly asserts that a *vis motrix* is measured by the force which will counteract it and keep the body to

which it is applied at rest, and the force thus used can hardly be other than a force measured statically; but his frequent insistence on the measure of *vis motrix* by the momentum which it causes shows that he had a conception also of the dynamical measure of force. He further supplies the measurement of mass as a fundamental quantity which is needed to establish the dynamical measure of force by calling attention to the possibility of comparing masses by means of the velocities given them when acted upon by equal forces.

Lagrange in the Mécanique Analytique gives the most explicit expression to the definition of force in general which is the bug-bear of so many thinkers, and which yet, after all, is the real expression of our belief about force, when he says: "We understand by force the cause, whatever it may be, which impresses or tends to impress a motion on a body to which we suppose it applied." He goes on to say: "It should be measured by the quantity of motion impressed or ready to be impressed. In the condition of equilibrium, the force produces no actual effect; it produces only a simple tendency to motion; but it should be measured by the effect which it would produce if it were not restrained from acting." Lagrange repeats this definition of the measure of force in the introduction to his Dynamics, when he says that the product of the mass and the accelerating force (Newton's *vis acceleratrix*) or the acceleration, expresses the motive force (Newton's *vis motrix*). I cannot find that Lagrange gives any definition of mass. From a statement in his treatment of centers of gravity it would seem that he considered the mass to be determined by its weight. He seems to endeavor to measure force in the purely dynamical way, without going into the matter as fully as he should for a complete elucidation of it.

Thomson and Tait say flatly that force is a direct object of sense, and define it as any cause which tends to alter a body's natural state of rest, or of uniform motion in a straight line. They assert that the measure of force is the quantity of motion which it produces per unit of time. They give no other definition of mass than the one given by Newton.

From the account which has been given of the views held or expressed by some of the great leaders of thought in matters of dynamics it is clear that very indefinite notions existed in their minds with respect not only to the proper definition of force, but even with respect to the proper measure of force, which is fundamental and necessary in the development of dynamics. The acute and valuable criticism by Mach of this fundamental notion is so colored in its expression by Mach's favorite principle of economy that it is not altogether satisfactory, and I accordingly shall attempt to present what seems to me the proper order of thought on this matter. Similar statements have been many times made, but there is still no general consent in the minds of physicists as to the statement which should be acceptable to everyone.

There is no doubt that the dynamical *measure* of force is the correct one to use in building up a system of units. The point of difference on which dispute arises is the order of precedence of the two concepts force and mass in the establishment of this definition. It is not uncommon to

have force defined as the product of mass by acceleration, or of mass by the acceleration which the mass would have if it were free to move. In this definition mass is the primary concept. Now, as I view the question, force is the primary concept, a direct object of sense, and we know it to be a cause of motion, or of the distortion of a body to which it is applied and which counteracts it when the distortion has reached a certain limit. In particular we know it as counteracting, or as being counteracted by, the weight of a body. This conception of force is adequate for the development of statics, in which we treat the principles of statics as statements of laws which are derived from experiment and confirmed by the proof that they are mutually consistent. Galileo's experiments on falling bodies are then the direct experimental proof in a limited case of the proportionality between the force which acts on a body, measured at any one place by a weight, and the acceleration imparted to the body. Newton's Second Law is a statement of Galileo's discovery, with this addition, that the acceleration imparted by a force is not the same for all bodies, but depends upon a certain characteristic of the body. This characteristic, the mass of the body, first calls for recognition at this point. In the view I have taken the mass is the factor of proportion between the force which acts on a body and the acceleration which it imparts to the body. Since we can measure forces by comparison with a standard force, we can also measure masses by the aid of properly instituted experiments. Whether we measure masses in this way or not, and it turns out to be not a satisfactory way to do it, we at least get from this relation between force, of which we have a concept, and motion, of which we have a concept, an adequate working concept of mass. Force is the primary concept and mass is a derived concept.

Now owing to the permanency of masses of matter it is convenient to construct our system of units with a mass as one of the fundamental units. We are able to do this and to compare one mass with another chosen as standard, without going through the operation of measuring forces, by utilizing the principle embodied in Newton's Third Law. This law asserts that bodies which interact, that is, which exert forces on each other, exert equal forces, and thus if the bodies are free to move, their accelerations will be inversely as their masses. By observation of the accelerations of two mutually interacting bodies we may thus compare their masses, and so construct a set or scale of masses, and use these masses and their accelerations to measure forces. Thus while the concept of force is primary in the order of thought, we may make the unit of mass fundamental in the development of a system of units.

The point upon which I wish to insist is that both reason and the history of mechanics show that the foundation of the science is the purely intuitional concept of force which is shared by every intelligent being, and that this intuitional concept is not only accurate so far as it goes but adequate to serve as the foundation of a great science. No use of the concept of force in the theories of physics has ever violated in any particular this original and intuitional concept of it. Even the brilliant endeavor of Hertz to found all the principles of dynamics upon the three

concepts of time, space and mass, cannot escape the criticism that the concept of mass is meaningless to us unless it is given to us by our experience of the inertia of matter when we exert force upon it. Once that concept is attained it may be used, as Hertz so beautifully used it, in the logical upbuilding of a system of dynamics. Perhaps my contention will be made clearer if we consider briefly the question whether it would be possible for us to construct our present system of dynamics if we were disembodied spirits, gifted with the means of observing spaces, times, and colors, but without the sensation of force. We could see colored volumes, sometimes moving with constant velocity, sometimes with varying velocity, and we could ascribe the changing velocity to the action of a force. We further could connect the force with the moving volume by setting it equal to the acceleration multiplied by some factor which we might name the mass. This equation would contain two unknown and unmeasured quantities, and would mean nothing unless we could go further. Now the advocates of the purely dynamical definition of the concept of force say that we can go further, by observing the mutual accelerations of two bodies and using these to obtain the ratio of their masses. If this can be done the matter is settled. But could it be done by the disembodied spirit? In our use of the mutual accelerations of two bodies to get their masses we must explicitly state that the bodies are arranged so as to interact (that is, to exert force on each other), and unless that condition is established the mutual accelerations of two bodies, however often repeated, can tell us nothing about their masses. A man at a station might observe two trains leaving the station in opposite directions with the same accelerations every day for ten years, and yet he could not compare their masses by any such observations. Eyes and mind only will not do it. To get the measure of mass we must start with the intuitional knowledge of force, and use it in the experiments by which we first define and then measure mass.

I now come to a much more difficult part of my subject, the consideration of the other primary concepts of space and time. Not many years ago we should have been willing to pass them over with a mere mention, admitting the impossibility of giving a definition or even an intelligible description of either of them, admitting the impossibility of determining an absolute or fixed point in space, or an absolute instant of time, but still asserting that we knew something about them both of which we were sure. At present we are driven by the development of the principle of relativity to examine anew the foundations of our thought in respect to these two primary concepts.

I suppose that the old ideas about space and time that have been of service to physicists since the beginning of the science are summed up as well as anywhere in Newton's words:

"Absolute and real time, the time of the mathematician, flows on equably, having no relation in itself or its nature to any external object. It is also called duration. Relative, apparent time, the time of common life, is an external measure of any duration cognized by the senses, by means of motion. It is commonly used in place of real time."

"Absolute space, having no relation in its nature to any external object, always remains alike everywhere and immovable. Relative space is the measure of this space, or any movable dimension, recognized by our senses as limited by its situation with respect to bodies. This is commonly thought of as equivalent to absolute space."

These definitions have been often justly criticized for the emphasis laid on the unfruitful ideas of absolute time and space. Perhaps the criticism has fallen rather upon Newton's subsequent expansion of his thought on these ideas. But do they not contain in the first place the conceptions of time and space which have been uniquely useful up to this time in physics, and in the second place, do they not contain what each one of us really thinks about time and space when he makes an honest examination of his knowledge? The essential feature of both these descriptions for our present purpose is Newton's declaration, both as to time and space, considered as species and not as magnitudes, that they are in themselves and in their nature without relation to any external object. It is this statement which is contradicted by some of the enunciations of the principle of relativity.

It is not necessary for me to give an account of the genesis of the principle of relativity. It may fairly be said to be based on the necessity of explaining the negative result of the famous experiment of Michelson and Morley, and on the convenience of being able to apply Maxwell's equations of the electromagnetic field without change of form to a system referred to moving axes. It is not needed to explain many of the remarkable results obtained by Fizeau, by Mascart, and by Brace, in the field of experimental optics, which to a first inspection seem to show that the earth and the medium around it through which light passes are relatively at rest, but which a closer study by Lorentz and others shows may be compatible with a reasonable theory of the structure of matter and the hypothesis that the luminiferous medium is at rest. It is also not needed to explain the dependence of the path of an electron in a field of crossed electric and magnetic forces upon its velocity, as exhibited in the beautiful experiments of Kauffmann and of Bucherer, for other theories in which the principle is not used lead to expressions for the path, which, for the present at least, are in as good accord with observation as those which are deduced by the aid of the principle of relativity.

There are two ways of presenting the principle of relativity. In the first way the principle is stated as a direct inductive conclusion from the experiment of Michelson and Morley, and asserts that so far as a conclusion can be drawn from that experiment and the others which have been tried to test the matter, there is no way by which the relative motion of the earth and the luminiferous medium can be determined from observations made on the passage of light when the source of light and the observer are moving with the earth. As thus presented the principle holds out as the object of future study the construction of a suitable theory of the structure of matter and of the luminiferous medium to account for this fundamental experiment as well as for all other known truths in the domains of light and electricity. If this theory is expressed

in terms of the Lorentz transformation, and thus shows a dependence of the measure of time and the measure of length upon the velocity of the system in which the observer is placed, it will further be the object of inquiry to construct a theory of the relations between the material of the system and the luminiferous medium which will account for the change in the units of length and in the motions of bodies by which the unit of time is determined. When I say to account for, I mean to describe in terms of force, time, and space, as we conceive those notions in our everyday experience, and as we use them in our ordinary physical work, so that the description when apprehended will be reduced to the lowest terms in which our thought about the universe can be expressed. Such a description is, as I view it, a real explanation, and surely it is not yet time to say that such an explanation is impossible.

The other way of presenting the principle of relativity consists in laying down as a fundamental postulate a general proposition expressing the hopelessness of any attempt to settle the question raised by the experiment of Michelson and Morley by any theory of the structure of the universe. This postulate sometimes assumes a formidable aspect, and involves more than the mere postulate of relativity. Thus Laue says: "The principle of relativity asserts that from the totality of natural phenomena we may, with continually increasing approximation, determine a system of reference, x, y, z, t, in which the laws of nature hold in a definite and mathematically simple form. This system of reference is by no means uniquely determined by the phenomena. There is rather a triple infinity of equally admissible systems, which move relatively to one another with uniform velocities."

The feature of this enunciation of the principle to which I referred as an addition to the principle is the expressed condition that in the system of reference the laws of nature hold in a definite and mathematically simple form. There is no warrant in the past history of physics for the adoption of such a postulate as that. Surely the history of the discovery of the so-called secondary laws of physics, such as Boyle's law, the laws of friction, the laws of polarization and of absorption of light, the laws of magnetization, and many others, will bear out the statement that in very many cases the first enunciation of the law is in a definite and mathematically simple form, and that further knowledge shows that this form is only a first approximation to the truth. Even in the case of such laws as the law of gravitation, or of electrical attraction and repulsion, from which we have not yet detected any deviation, does anyone dare to say that they are universally true for all bodies and at all distances? Can we even feel sure that Maxwell's electromagnetic equations hold true with absolute exactness? They need supplementing when they are applied to material bodies. Can we be sure that they hold without modification, in rapidly moving bodies, or at extremely minute distances in free space? Or, from another point of view, admitting that the object of physical study is to reduce the description of natural phenomena to a set of simple laws, have we a right to assume that, in our analysis of the structure of matter and of the luminiferous medium, we have as yet

reached the ultimate model in which such simple laws will be operative? The answer to this question must be a negative one. Yet it is surely true that if it were not for this demand of simplicity, immediately attainable and at present expressed in the electromagnetic equations, the chief incentive to the development of the theory of relativity would be wanting.

But this is not the heart of the matter. With the principle of relativity as a basal postulate, not expressing our present inability but rather the hopelessness of any attempt to obtain ability, a complete description has been given of the phenomena now known to physicists, at least in the domains of mechanics, light and electricity. The difficulty which I find in accepting the principle, with the universality that is predicated of it, is that it does so much more than this.

The theories of J. J. Thomson and of Lorentz made physicists familiar with the notion of electrical mass, exhibited by the variability of the mass of a moving charged body, or by the apparent variable inertia of a moving charge expressed as a function of its velocity, and further with the notion that as the velocity of the charge approaches the velocity of light, the magnitude of the electrical mass approaches infinity, so that the velocity of an electrical charge, of an electron, and therefore presumably of matter, if it is entirely electrical in its structure, can never surpass the velocity of light. In these theories this remarkable conclusion was explained by the interaction between the moving charge and the ether. In the theory of relativity the same conclusion is reached as the consequence of a purely kinematical theorem, giving the rule for the addition of velocities, and not only does it hold for real moving charges, but for any action whatever which is conceivably transmitted through space. In particular this finite velocity of transmission must be ascribed to gravitational action. Now the Newtonian theory of gravitation assumes a practically infinite velocity of transmission of gravitational action, and astronomical observations have never given any warrant for the belief that its velocity of transmission is even of the order of magnitude of the velocity of light. The attempt has been made to reconcile the theory of relativity with the observed motions of the planets by the adoption of an arbitrarily chosen term in the formula for the force on a planet to represent what is equivalent to a counteracting force to annul the tangential acceleration which would arise from the finite rate of transmission of gravitational force. This is manifestly an artifice and not an explanation. If the principle of relativity is of universal application, it should not need the introduction of such an artifice to help it out in the solution of one of the classical problems of physics.

Furthermore, the principle of relativity in this metaphysical form professes to be able to abandon the hypothesis of an ether. All the necessary descriptions of the crucial experiments in optics and electricity by which the theories of the universe are now being tested can be given without the use of that hypothesis. Indeed the principle asserts our inability even to determine any one frame of reference that can be distinguished from another, or, what means the same thing, to detect any

relative motion of the earth and the ether, and so to ascribe to the ether any sort of motion; from which it is concluded that the philosophical course is to abandon the concept of the ether altogether. This question will be amply and ably discussed this morning, but I may venture to say that in my opinion the abandonment of the hypothesis of an ether at the present time is a great and serious retrograde step in the development of speculative physics. The principle of relativity accounts for the negative result of the experiment of Michelson and Morley, but without an ether how do we account for the interference phenomena which made that experiment possible? There are only two ways yet thought of to account for the passage of light through space. Are the supporters of the theory of relativity going to return to the corpuscles of Newton? Are they willing to explain the colors of thin plates by invoking "the fits of easy reflection and of easy transmission"? Are they satisfied to say about diffraction that the corpuscles near an obstacle "move backwards and forwards with a motion like that of an eel"? How are they going to explain the plain facts of optics? Presumably they are postponing this necessary business until the consequences of the principle of relativity have been worked out. Perhaps there is some other conceivable mode of connection between bodies, by means of which periodic disturbances can be transmitted. We may imagine a sort of tentacular ether stretching like strings from electron to electron, serving as physical lines of force, and transmitting waves as a vibrating string does. Such a luminiferous medium would not meet the postulate of simplicity, but it conceivably might work. But whatever the properties of the medium may be, there is choice only between corpuscles and a medium, and I submit that it is incumbent upon the advocates of the new views to propose and develop an explanation of the transmission of light and of the phenomena which have been interpreted for so long as demonstrating its periodicity. Otherwise they are asking us to abandon what has furnished a sound basis for the interpretation of phenomena and for constructive work in order to preserve the universality of a metaphysical postulate.

The electromagnetic equations, too, the retention of which in their present simple form is the *sine qua non* of the promoters of the principle of relativity, were not only developed by the conscious use of the hypothesis of a medium in which the electric and magnetic forces exist, but can be interpreted intelligibly only in terms of some such medium. The abandonment of this hypothesis reminds one of Baron Munchausen's feat performed while he was making his escape from prison. Since your historical reading may not have extended to the autobiography of this famous man, I may be permitted to relate that the Baron was letting himself down from the windows of a high town by a rope, and when he reached the end of it he found that he still had a long distance to go. The last part of the descent was particularly difficult, so to get rope enough he ingeniously spliced on an additional piece, which he obtained by cutting off the part above him.

The principle of relativity in its metaphysical form ignores the ac-

celerations of bodies. It is true that the experimental results to which the principle has been applied with such success are such that the study of acceleration in terms of the theory of relativity has not become necessary. But is it not reasonable to suppose that when suitable experiments have been invented and tried to test the effect of the acceleration of a system on the progress of light in it, it may be found that an effect can be detected? Some effect may be detected, for example, due to the rotation of a body. I have never been able to perceive any sound objection to Newton's assertion that we have evidence of absolute rotation by the observation of centrifugal force, and if a fixed direction of an axis and an absolute velocity of rotation can be determined in a mechanical system when accelerations are taken into consideration, why should the principle of relativity be treated as having universal validity?

But, after all, these questions raised by the development of the principle of relativity are of secondary importance. The central question is whether or not this principle can ever furnish a satisfactory explanation of natural phenomena. The formulas derived from it are evidently merely descriptive. This may be said with truth about all the formulas in which the general theories of physics have been embodied. Kirchhoff designates as the task of the science of mechanics, the description of the motions which occur in nature completely and in the simplest possible way. This assertion that the task of the theoretical physicist is done when he has reduced the phenomena with which he is dealing to a set of formulas, or, as we may say, when he has constructed an ideal model which will produce the phenomena, is one to which we would all assent in general. At the same time most of us would reserve the right to criticize each model thus presented, and to give to one or the other a preference based on considerations which are not necessarily limited to the simplicity of the model or to the completeness with which it reproduces the phenomena. Surely an additional test of the value of the model will be the intelligibility of the elements of which it is composed.

This last test has been generally met in the models which have been proposed as descriptions of natural phenomena. We can understand from what we see and feel what is meant by the motions of elastic spheres, and the model which uses them to represent the behavior of a gas is not only competent to reproduce the behavior of a gas but is intelligible in the elements of which it is composed. The model of the elastic solid ether, incomplete and objectionable as it became when the subject of optics was enlarged and developed, was intelligible in its elements. The model of electromagnetic operations embodied in Maxwell's formulas is also one which is thus intelligible in its elements. When I say this I do not mean that we know all about electric and magnetic forces, but I mean that we do know enough about such forces to have a clear notion of their variation in space and their variation in time.

This feature of the ideal model or description seems to me to be necessary in order to make the model acceptable as the ultimate or last attainable explanation of phenomena. The elements of which the model is constructed must be of types which are immediately perceived by the

senses and which are accepted by everybody as the ultimate data of consciousness. It is only out of such elements that an explanation, in distinction from a mere barren set of formulas, can be constructed. A description of phenomena in terms of four dimensions in space would be unsatisfactory to me as an explanation because by no stretch of my imagination can I make myself believe in the reality of a fourth dimension. The description of phenomena in terms of a time which is a function of the velocity of the body on which I reside will be, I fear, equally unsatisfactory to me because, try I ever so hard, I cannot make myself realize that such a time is conceivable.

Tried by this test I feel that the principle of relativity does not speak the final word in the discussion about the structure of the universe. The formulas which flow from it may be in complete accord with all discovered truth, but they are expressed in terms which themselves are not in harmony with my ultimate notions about space and time. That this is true is so evident that it is generally admitted. Some writers say that we should not let this circumstance disturb us, because Kant has said that time and space are mere forms of perception, a scheme in which we must arrange occurrences so that they may acquire objective significance. I do not altogether understand what Kant meant by this, but I am sure he did not mean that by the exercise of our wills we can violently eject from our consciousness the notions of space and time which we have in common with the whole race of man, and impose on ourselves other and radically different notions. Planck compares our position before the new notions presented by the theory of relativity to the position of the medieval peoples before the notion of the antipodes. It seems to me that there is no real similarity between the two positions. Many men in the Middle Ages believed that there were no antipodes, but their belief was based on reasons, and so far were they from being unable to conceive of antipodes and to believe in their existence, that there were men who actually maintained their existence, and were pursued therefor as heretics. I do not believe that there is any man now living who can assert with truth that he can conceive a time which is a function of velocity or is willing to go to the stake for the conviction that his "now" is another man's "future" and still another man's "past."

One of the members of this Society, recognizing our present inability to conceive of relative time, and conceiving our intuitions of space and time to be the result of heredity operating through many generations of men who lacked the light of relativity, once proposed to me that everyone who could get even a glimmer of the notion of relative time should persistently exercise his mind therein and teach it to his students, in the hope that in a few generations the notion would emerge with the force of an intuition. It would not be fair to leave the impression that he was solemnly serious when he made this suggestion. When Matthew Arnold was asked to endure the transliteration of Greek names into English in order that the new forms might become familiar to future scholars, he answered that he was not willing to spend his days in a wilderness of pedantry that his children might enjoy an orthographical Canaan; and

mutatis mutandis the same answer may be given in this case. But a more pertinent answer is, to my mind, this, that the attempt to reorganize the perceptions of the human mind in respect to space and time is doomed to failure. "Which of you by taking thought can add one cubit unto his stature?" I believe that these ultimate perceptions are the same for all men now, have been the same for all men in the past, and will be the same for all men in the future. I believe further that this is true because the universe has a real existence apart from our perceptions of it, and that through its relations to our minds it imposes upon us certain common elementary notions which are true and shared by everybody.

Therefore, from my point of view, I cannot see in the principle of relativity the ultimate solution of the problem of the universe. A solution to be really serviceable must be intelligible to everybody, to the common man as well as to the trained scholar. All previous physical theories have been thus intelligible. Can we venture to believe that the new space and time introduced by the principle of relativity are either thus intelligible now or will become so hereafter? A theory becomes intelligible when it is expressed in terms of the primary concepts of force, space, and time, as they are understood by the whole race of man. When a physical law is expressed in terms of those concepts we feel that we have a reason for it, we rest intellectually satisfied on the ultimate basis of immediate knowledge. Have we not a right to ask of those leaders of thought to whom we owe the development of the theory of relativity, that they recognize the limited and partial applicability of that theory and its inability to describe the universe in intelligible terms, and to exhort them to pursue their brilliant course until they succeed in explaining the principle of relativity by reducing it to a mode of action expressed in terms of the primary concepts of physics?

THE KERR ROTATION FOR TRANSVERSE MAGNETIC FIELDS.[1]

BY L. R. INGERSOLL.

THE older magneto-optic theory did not anticipate any effect of a magnetic field on light reflected from the surface of a magnetic metal, if the lines of force were normal to the plane of incidence. Some years ago, however, C. H. Wind predicted that an effect should exist for this case although it would be very small, and Zeeman was able to observe it for the most favorable case, viz., steel at 75 degrees incidence.

The author has recently obtained some results by the bolometric method (see *Astrophys. Jour.*, Nov., 1910) which seem to bear out Wind's theory to a considerable extent. The effect observed is a slight change in the azimuth of "restored polarization" of the reflected light, due to a magnetic field which is parallel to the surface and perpendicular to the plane of incidence of the light. The maximum value of this change is about 17 minutes of arc for steel, and smaller for the other metals. Both steel and cobalt give curves closely approximating Wind's computed

[1] Abstract of a paper presented at the Chicago meeting of the Physical Society, Dec. 2, 1911.

curves, the effect increasing from zero at normal incidence to a positive maximum at about 70 degrees, changing to a negative maximum, and becoming zero again at grazing incidence. Nickel on the other hand does not even approximate the theoretical curve but gives values opposite in sign to the other metals. This is similar to its action in the case of the Hall effect, and undoubtedly these two phenomena are r lated.

The author is also able to show that the only component of vibration effective in producing this effect is the one whose electric vector is in the plane of incidence. This is in agreement with theory.

A New Type of Apparatus for Measuring Linear Expansion.[1]

By Arthur W. Gray.

THE bar to be measured is supported horizontally in an electrically heated tubular furnace designed especially to secure uniformity of temperature. The tube, of metal and wound with resistance ribbon, has its central portion completely filled by a block of metal pierced by two longitudinal cavities, of square cross-section, symmetrically situated above and below the geometrical axis of the tube. The lower cavity is almost completely filled by the bar to be measured, while the upper one contains a similar dummy bar whose temperature is determined by an electric thermometer placed within a hole following its axis. Properly placed differential thermoelements indicate the distribution of temperature both radially and longitudinally. Heat losses through the ends are prevented by long plugs, each consisting of two thick, solid cylinders of metal united by a thin circumferential tube and separated by a thick layer of heat insulating material. An independent electrical heating coil wound in a groove within the outside cylinder of each plug permits the end to be maintained at a temperature almost the same as that of the interior, thus reducing the longitudinal temperature gradient and thereby the outward conduction of heat. The solid metallic cylinders prevent loss by radiation from the interior, and at the same time aid in producing uniformity of temperature perpendicular to the axis of the tube.

Changes in the length of the lower bar are measured by micrometer microscopes focused on fine wires freely suspended over the ends of the bar and stretched vertically by the weight of vanes immersed in oil, whose viscosity is adjusted to damp any swinging of the wires so that their motions will be almost, but not quite, aperiodic. Disturbances from changes in level are avoided by grinding the ends to form portions of a horizontal cylinder coaxial with the center of the bar. An unheated bar, likewise supporting suspended wires, forms a reference standard for determining any changes that might occur in the distance between the axes of the microscopes. The reference bar is placed below the furnace, the plane of its wires intersecting that of the test bar's wires in a vertical passing through the centers of both bars.

The microscopes are rigidly united by a horizontal slab of marble rotatable

[1] Abstract of a paper presented at the Washington meeting of the Physical Society, Dec. 27–30, 1911.

about the vertical containing the centers of both bars, and provided with the necessary adjustments for rapid focusing.

BUREAU OF STANDARDS,
WASHINGTON, D. C.,
December 9, 1911.

THE SPECTRUM OF TITANIUM IN A PARTIAL VACUUM AND THE PROPORTIONALITY OF DISPLACEMENT TO PRESSURE AT MODERATE PRESSURES.[1]

BY WALTER S. ADAMS AND HENRY G. GALE.

IN an earlier communication we have called attention to the fact that the enhanced lines are not easily reversed, and have referred briefly to the possible bearing of this fact on the spectrum of the chromosphere. On some photographs taken at a pressure of about 10 centimeters we have found the enhanced lines considerably strengthened over what they are at atmospheric pressure. This fact may well account for the strength of the enhanced lines in the upper chromosphere, since the pressures are no doubt low in the higher levels of the solar atmosphere.

The shift of a selected list of 20 titanium lines has been measured at 16, 12, 8, 6, 4 and 2 atmospheres above normal pressure, and at a pressure of 10 centimeters.

The twenty lines selected were all between λ 4300 and λ 4600. They were all reversed and well adapted for measurement. We have used the mean displacement of the twenty lines at each pressure in investigating the linearity of the displacement-pressure law. If we assume a linear relationship and deduce the mean value by a least square solution the following residuals, expressed in Ångströms, are obtained for the observed minus the computed mean shift at each pressure.

P.	Obs.-Comp.	Shift per Atm.
2	+0.0005	0.0042
4	−0.0003	0.0040
6	+0.0003	0.0042
8	−0.0026	0.0038
12	+0.0004	0.0042
16	+0.0028	0.0043
		Mean 0.0041

Rossi gives 0.0045 for the average displacement per atmosphere between 15 and 100 atmospheres for 16 of these same lines. Two plates at a pressure less than 1 atmosphere, give a mean shift of 0.005 Ångströms per atmosphere.

The photographs were taken in the laboratory of the Mount Wilson Solar Observatory. The second order of a 7-inch Michelson grating was used in the 30-foot vertical spectrograph. We are indebted to Miss Lasby for many of the measurements.

[1]Abstract of a paper presented at the Chicago meeting of the Physical Society, Dec. 2, 1911.

The Effect of Temperature on the Absorbed Charge in Electric Condensers.[1]

By Anthony Zeleny.

THE values of the absorbed charge obtained from a paper condenser were determined at various temperatures between $-20°$ and $+22°$ C. The plotted curve shows a sudden change in curvature at and below 0° C., the average temperature coefficient above that point being 0.136, while below it is only 0.037. This change at the freezing point of water suggests that a large part of the absorbed charge may be due to the presence of moisture in the paraffin. The condenser used was made without boiling in vacuo. The cause of this sudden change in curvature will be investigated.

The Electrical Resistance and the Polarization E.M.F. of a Mixture of Clay, Feldspar and Quartz.[1]

By A. A. Somerville and O. E. Buckley.

POTTERY samples containing English ball clay, feldspar and quartz were moulded in the form of rods about one cm. in diameter and three to five cm. in length and baked at a temperature of 1100° C. Electrical connections were made by passing nickel wires through holes near each end of a rod. Such a piece of pottery is an insulator at ordinary temperatures but becomes a conductor at higher temperatures. It does not follow the most simple form of Ohm's law, $I = E/R$, but due to something like a polarization effect this becomes a conductor to which the form of the equation, $I = (E - e)/R$, is applicable. e is eliminated and R is measured by using an alternating current device, a sechometer, in connection with a dry cell and wheatstone bridge. Temperatures are measured by means of a platinum resistance thermometer and indicacted on an automatic recorder. The heating is accomplished by means of a tubular resistance furnace.

The resistance of one of these rods of pottery behaves much like that of glass, porcelain, quartz and the Nernst glower. It is near 10,000,000 ohms at about 550° C. and decreases rapidly as temperature increases until at 1100° C. it measures approximately 15,000 ohms.

When a direct current is passed through a piece of the pottery a polarization or counter E.M.F. is produced which is represented in the above equation by e. The specimen is charged for a given time, at a definite temperature, by a certain voltage. A switch is then thrown to connect it into a potentiometer circuit and a balance obtained giving the value of e as quickly as possible, usually after only a few seconds. The time-decay curve of e is then determined. During the whole experiment the piece of pottery is suspended by means of its lead wires in the furnace. Nickel leads have been used.

It is found that e is a function of the charging voltage, time of charging and temperature.

[1] Abstract of a paper presented at the Washington meeting of the Physical Society, Dec. 27–30, 1911.

1. e increases rapidly as the charging voltage goes up to about 6 or 8 volts and then slowly as the latter moves up to 110 volts.

2. e increases rapidly with time up to about 2 minutes and thereafter slowly.

3. e decreases as temperature increases.

The maximum polarization or counter E.M.F. obtained is about 2 volts.

The rate of discharge is dependent on temperature, time of charging and material and size of electrodes, leads or connectors used. The rate of discharge increases as temperature increases and decreases as time of charging increases. Rate of discharge also decreases as the size or number of nickel wires used as leads are increased. If nichrome, an alloy presumably containing nickel, is used the e obtained is the same as for nickel but in the case of the alloy the decay or discharge is much more rapid. If iron leads are used the e obtained is slightly higher than is the case when using nickel and the discharge also more rapid.

ABSORPTION OF THE β-PARTICLES BY GASES.[1]

BY ALOIS F. KOVARIK.

THIS investigation has for its purpose: 1. To obtain a relation between the activites of the soft and hard β radiations as, *e. g.*, from RaD and RaE, Actinium B and Actinium D, Thorium B and Thorium E, etc. This is difficult when the absorbing material used is a metal foil.

2. To determine the coefficient of absorption of the β-particles in terms of gases.

3. To determine the number of soft β-particles emitted from one disintegrating atom which emits soft β-particles.

This preliminary work was done with RaD +E + F. The apparatus consists of an ionization vessel 2 mm. deep placed in a cylinder in which the pressure can be run up to about 20 atmospheres. The RaE β-particles are absorbed exponentially in air and CO_2 down to about 10 per cent. of the initial value. The value of μ for RaE rays is 0.0149 cm^{-1} air and 0.0296 cm^{-1} CO_2 (atmospheric pressure).

The RaD β-particles are undoubtedly of two kinds of penetrability. This was already shown by O. v. Beyer, Hahn and Meitner, *Phys. Z. S.*, 12 Jg., 378, 1911. The activity of the harder RaD β-particles, when reduced to no absorption, is about 20 per cent. of the activity of RaE rays and the value of μ for these rays is 0.094 cm^{-1} air and 0.181 cm^{-1} CO_2. The softer rays have a much larger value of μ and their activity appears to be several times that of RaE rays, reduced to no absorption.

Eve,[2] who recently published values for the coefficient of absorption by air of the β-rays from RaC, obtained by an entirely different method, got for the hard rays $\mu = 0.0045$ cm^{-1} air. In case of aluminum these rays have a value of $\mu = 13.5$ cm^{-1} Al.

The value of μ for RaE rays is 43.5 cm^{-1} Al. It will be noticed that the relation between the values of μ in terms of aluminum and air for the rays from

[1] Abstract of a paper presented at the Washington meeting of the Physical Society, Dec. 27–30, 1911.

[2] Eve, A. S., Phil. Mag., July, 1911.

RaE, as above, and RaC, as found by Eve, is about the same. If this relation holds for RaD rays then the value of μ for the harder β-particles from RaD is 270 cm^{-1} aluminum.

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THE SPECTRA OF IRON AND TITANIUM AT MODERATE PRESSURES.[1]

BY HENRY G. GALE AND WALTER S. ADAMS.

AN investigation of the arc spectrum of iron at a pressure of 9 atmospheres, and of the arc and spark spectra of titanium at pressures ranging from a partial vacuum to 17 atmospheres, has been completely by us at the Mount Wilson Solar Observatory.

The range of wave-length included in our photographs extended from λ 3600 to λ 6700. A few plates were also taken for the calcium triplets at λ 3965 and at λ 6192. A total of 250 plates were taken, with from two to eight exposures on a plate. The scale of the plates was approximately 1 mm. per Ångström. A strip of spectrum about 400 Ångströms long was usually measurable on each plate.

The following results have been obtained:

1. Reversal is a function of wave-length, being most frequent in the more refrangible part of the spectrum, and becoming less so toward longer wave-lengths. A similar result was announced by Hale in 1902.

2. The enhanced lines of titanium are not reversed in the arc, and only very faintly in the spark, except in the ultra-violet, even at a pressure of 17 atmospheres. This suggests an explanation of the prominence of the enhanced lines in the lower levels of the chromospheric spectrum.

3. At a pressure of 10 cm. the enhanced lines of titanium are relatively strengthened with respect to the arc lines. This suggests an explanation of the prominence of the enhanced lines in the upper levels of the chromosphere, where the pressure is undoubtedly low.

4. The displacements of the titanium arc lines are found to be accurately proportional to the pressure at pressures of 2, 4, 6, 8, 12 and 16 atmospheres above atmospheric pressure. The proportionality holds also at pressures less than one atmosphere.

5. The 23 enhanced lines of titanium measured in the arc show an average displacement, by a pressure of 8 atmospheres, of 0.034 Ångström. The 103 other lines in the same region of wave-lengths have an average displacement of 0.023 Ångström. On the spark plates the 15 enhanced lines have an average displacement of 0.050 and the 71 arc lines in the same region an average displacement of 0.028. *On the average*, then, the enhanced lines have a displacement which is about 50 per cent. larger than that of the arc lines. A few of the enhanced lines, however, have small displacements.

6. As shown by the figures above, the displacements in the spark are, on the

[1] Abstract of a paper presented at the Washington meeting of the Physical Society, Dec. 27–30, 1911.

average, greater than in the arc. This difference is most pronounced in the case of the enhanced lines, but is in all probability true for the arc lines as well. In the selected list of 20 arc lines used for testing the proportionality of displacement to pressure, which were measured on many plates in both arc and spark, every line is shifted more in the spark than in the arc. The average difference $\Delta_{spark} - \Delta_{arc} = 0.007$ Ångström. For all the enhanced lines measured in both arc and spark the mean difference is 0.014 Ångström.

7. We have been unable to detect any change in displacement with changes in capacity, self-induction, spark gap or current strength, or with a change from air to carbon dioxide.

8. The low temperature or "flame" lines of iron appear to form a distinct group and have small displacements under pressure.

9. There are three other easily recognizable groups in the iron spectrum, for which the ratios of the mean displacements are 1 : 2.3 : 4.5. For the four groups the ratios are 1 : 1.5 : 3.4 : 6.6. If the two groups of smallest displacement are united we have 1 : 2.5 : 4.9 instead of 1 : 2 : 4 as announced by Duffield and by King.

10. The values of the average displacement for the four iron groups at different wave-lengths are well represented by a law of variation of the displacement with the third power of the wave-length.

11. We have been unable to distinguish separate groups in the titanium spectrum, but omitting the enhanced lines, and a few lines of very large shift, similar to the *d* group in iron, the shifts appear to be represented most nearly by a law involving the square of the wave-length.

12. The calcium lines measured in the triplets of the second subordinate series at λ 3965 and at λ 6129 give shifts which are represented by a law involving the first power of the wave-length.

13. There may be some significance in the fact that calcium, titanium and iron occur respectively in the second, fourth and eighth groups of the Mendeléjeff table.

14. In the spectrum of iron there appears to be a direct relationship between pressure shift and Zeeman effect, for lines of the same group, and of the same type of magnetic separation.

15. We have found no evidence to confirm the results of Duffie d and King, that, at a given pressure the displacement may have different values for the same source.

16. A number of plates of the titanium arc under a pressure of 5 atmospheres (total) of illuminating gas were taken. There was no appreciable difference in the shift of the 16 arc lines measured, but the 7 enhanced lines showed an average displacement of 0.006 Ångström greater in hydrogen (illuminating gas) than in carbon dioxide.

17. As is well known, the enhanced lines are relatively strengthened in a hydrogen atmosphere. This holds at a pressure of 5 atmospheres as well as at one atmosphere.

18. The small shift of the flame lines of iron may be a property of the flame lines of iron alone. The low temperature line, 4227, of calcium appears to have a shift almost exactly twice that of H and K. The lines of titanium which are strengthened in sunspots, however, appear on the average to have small shifts.

A Quantitative Measure of Development in Scientific Observation.[1]

By Otto Stuhlmann, Jr.

CURVES have been obtained showing the progress in development of accuracy in scientific observation for individuals and for a class of students in a course in laboratory physics.

The errors in an inexperienced observer's results were found to vary periodically but decrease in magnitude as the familiarity with the particular experiment increased. These positive and negative errors were found to be equally probable and if plotted against the trial in which they occurred, it was found that the curve followed very closely the damped oscillatory equation

$$E = ke^{-\alpha t} \sin pt.$$

It shows that the maximum error (E) in either direction decays away in geometric progression as the trials (t) increased in arithmetical progression. Here the logarithmic decrement,

$$\log \frac{E_1}{E_2} = \frac{\alpha T}{2} \equiv \delta,$$

or damping of the oscillations per half period, can be shown to be the measure of the development in scientific observation of the subject.

Further curves were obtained for advanced students and for an expert experimenter. They show that while the initial errors, in the results from an unfamiliar experiment, are relatively large for all classes of observers, that the greater the experience in scientific observation the more rapid the damping. The curve eventually deteriorating as in the expert observer's curves into the type generally known as the "dead beat curve,"

$$E = Ae^{\alpha_1 t} + Be^{\alpha_2 t}.$$

This latter type is commonly known to scientific observers as the "personal equation." Hence the "personal equation" is a curve originally of the form of the first equation but changing gradually to the "dead beat" type as the experience of the subject and hence his accuracy of observation increases.

The same is also here applied to the progress in observation to a class of students in the physical laboratory, although the class as a whole never reaching the "dead beat" type of observational curve.

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Relation between the Joule Effect and the Induction and Permeability in the Same Specimens of Steel.[1]

By S. R. Williams.

A SURVEY of the investigations which have been carried out on the varied phenomena of ferromagnetism indicates that too little comparative work has been accomplished, in which one phenomenon has been compared with another in the same specimen of ferromagnetic substance.

[1] Abstract of a paper presented at the Washington meeting of the Physical Society, Dec. 27–30, 1911.

I have therefore made a collection of rods (80 cm. in length) of ferromagnetic substances in which I propose to study the various magnetic effects possible to be carried out with specimens of this shape. Among these various phenomena may be mentioned the Joule and Wiedemann magnetostrictive effects, the permeability, susceptibility, carbon content, rigidity, crystalline structure by micro-photography, the Kerr effect, etc. From a comparative study on such a group of rods it is hoped that something more definite may be arrived at regarding these inter-relations.

The present paper deals with the relation between the Joule magnetostrictive effect and the induction and permeability of the same rods. The first part deals with the Joule effect and a photographic method of quickly registering the magnetic changes in length with corresponding field strengths. Heat effects are thus eliminated by obtaining the magnetic changes before the temperature changes occur. The second part is a study of the relative position of maximum permeability and maximum increase in length as regards field strength. Accompanying this are micro-photographs of the specimens and analysis showing carbon content.

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THE APPLICATION OF STATISTICAL PRINCIPLES TO PHOTOELECTRIC EFFECTS AND SOME ALLIED PHENOMENA.[1]

BY O. W. RICHARDSON.

THE chief part of this paper deals with the application of statistical principles and of thermodynamics to the reflection of electrons by bodies and the emission of electrons under the influence of photoelectric action. Considering first of all the case of a body which is devoid of photoelectric activity, the group of electrons which arrive at the surface from outside is balanced, in the steady state, by the reflected electrons together with those liberated by thermionic emission. It is shown that the distribution of velocity among the reflected electrons is in accordance with Maxwell's law. Moreover if $F(u \cdot v \cdot w \cdot u_1)du_1$ denotes the proportion of the electrons reflected, out of the incident group having velocity components between u and $u + du$, v and $v + dv$ and w and $w + dw$, which have their u component of velocity between u_1 and $u_1 + du_1$ then F has to satisfy the equation

$$\int_0^\infty \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} ue^{-km(u^2+v^2+w^2)} \{ F(u \cdot v \cdot w \cdot u_1) - 2kmu_1 e^{-kmu_1^2} \} dudvdw = 0. \quad (1)$$

There are similar equations for the functions which determine the proportions of the other components v_1 w_1.

The more general problem of the exchange of energy and electricity under the combined influence of ethereal radiation, thermionic emission and photoelectric action, in the state of statistical equilibrium, which characterizes an enclosed system at a uniform temperature, is then considered. By a slight modification of the usual proofs it is shown that the complete (black body) ethereal radiation has the proper-

[1] Abstract of a paper presented at the Washington meeting of the Physical Society, Dec. 27-30, 1911.

ties which have hitherto been deduced for it. It is shown that the distribution of kinetic energy among the electrons in the complete atmosphere is in accordance with Maxwell's law and the average number n of electrons per unit volume at any point is connected with the local value w of the potential energy of an electron by the relation.

$$RT \log n + w = C, \tag{2}$$

where C is a function of T only. It is also shown that in the case of a cavity in the interior of any particular substance n is given by

$$n = Ae^{\int \frac{\phi}{RT^2} dT}, \tag{3}$$

where A is a constant characteristic of the material, but independent of T, and φ is the absorption of heat which is associated with the liberation of one electron at the surface. The above relations apply to the complete atmosphere of electrons and do not distinguish between their modes of origin. The electrons originate, in general, from the combined effects of reflection, thermionic emission and photoelectric action. It is clear that there is a very close correspondence between the thermionic emission, and the photoelectric emission arising from the complete ethereal radiation characteristic of any given temperature. It is pointed out, however, that the experimental evidence is against the view that the thermionic emission is nothing more than the aggregate photoelectric effect of the complete ethereal (temperature) radiation.

The properties of an ideal body which is photoelectrically active but which is devoid of thermionic emissivity and of the power of reflecting electrons is then discussed. It is shown that the photoelectric emission from such a body must satisfy the equation

$$\frac{c}{4\pi} \int_0^\infty \epsilon F(\nu T) E(\nu) d\nu = A \left(\frac{R}{2m\pi} \right)^{\frac{1}{2}} T^{\frac{1}{2}} e^{\int \frac{\phi}{RT^2} dT} \tag{4}$$

for all values of the temperature T. Here $E(\nu)d\nu$ is the density of the black body radiation of frequency between ν and $\nu + d\nu$, ϵ is the proportion of it which is absorbed (*i. e.*, not reflected), $F(\nu T)$ is the number of electrons emitted by photoelectric action when unit quantity of energy of this frequency is absorbed at this temperature. A formal solution of (4) under the restrictions that $\epsilon F(\nu T)$, A and φ are independent of T, is given, but it appears to be divergent except when ν is very large. It is shown that equation (4) is consistent with the view suggested by Einstein that the light of frequency ν communicates to the electrons an amount of energy $h\nu$, where h is Planck's constant, that part of this is used up by the work φ required to liberate an electron and that the rest appears as the kinetic energy of the electrons which have escaped. What appears to be a rather more natural view, however, leads to the conclusion that only one fourth of the energy of the escaped electrons arises in this way, the remainder being the energy they possessed when inside the metal. This view is also consistent with equation (4). It is also shown that the distribution of velocity among the electrons emitted under the influence

of radiation of frequency ν, incident in the altitude between θ and $\theta + d\theta$ and azimuth φ and $\varphi + d\varphi$ has to satisfy equations of the type:

$$\int_0^{\pi/2}\int_0^{2\pi}\int^{\infty} \sin\theta\cos\theta E(\nu)\{\epsilon_1(\nu\theta)F_1(\nu\theta)[f_1(\nu\cdot\theta\cdot u\cdot v\cdot w\cdot k) - 2kmue^{-kmu^2}] + \epsilon_2(\nu\theta)F_2(\nu\theta)[f_2(\nu\cdot\theta\cdot u\cdot v\cdot w\cdot k) - 2kmue^{-kmu^2}]\}d\theta d\varphi d\nu = 0. \tag{5}$$

Here the suffixes 1 and 2 refer to radiation polarized in and perpendicular to the plane of incidence respectively. ϵ and F are the same as before except for the restriction as to direction. $f(\nu\cdot\theta\cdot u\cdot v\cdot w\cdot k)$ denote the proportion of the liberated electrons which have velocity components between u and $u + du$, u being normal to the surface of emission. The temperature is $T = (2kR)^{-1}$. There are similar equations for the other components $v\cdot w$. All are true for all values of $u\cdot v\cdot w$ and k.

The consequences of restricting the discussion to the case of an ideal body, which is photoelectrically active, but devoid of thermionic emissivity, and which reflects none of the electrons which reach its surface, are considered. It is shown that in the case of a real body the group of electrons which is made up of the sum of those reflected and those emitted photoelectrically will have Maxwell's distribution of velocity. The writer has not been able to show that this is true of the two subdivisions separately but since reflection and photoelectric emission are independent acts it follows that it will be very nearly, if not exactly, true for each subdivision. Thus the photoelectric properties which have been deduced for the ideal matter will possibly be exactly true and probably be approximately true for real matter.

The paper also contains a discussion of the theory of thermoelectric effects from a wider standpoint than that of the former paper on this subject (PHYS. REV., Vol. 33, p. 448, 1911). It is shown that, except when the internal free electrons can all be considered to have very nearly the same potential energy, and to be capable of occupying nearly the whole volume of the material, it is necessary to replace $\log n$ in the earlier formulæ by $\int n \log n d\tau \div \int n d\tau$, where n is the number which expresses, per unit volume, the local value of the sum of both the electrons which are actually free and those which may become free under the dynamical actions actually occurring. In general, thermoelectric phenomena do not distinguish between the actually free and the temporarily bound electrons. In particular, no very close connection is to be expected between thermoelectric effects and changes in electrical conductivity (for example at the melting point).

The part possibly played in electric conduction by electrons similarly situated to those which give rise to polarization in dielectrics is considered. It is shown that some of the peculiar features exhibited by galvanomagnetic phenomena might arise in this way but it does not appear that all the characteristics of this very complex group of effects can be accounted for by such a simple hypothesis.

The very high electrical conductivity of metals at the very lowest temperatures, observed by Kammerlingh Onnes, would appear to indicate that either the number

of the centers of force considered in the previous paper (PHYS. REV. vol. 34, p. 77, 1912), or their strength, is greatly diminished at these very low temperatures.

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A STUDY OF CRYSTAL RECTIFIERS.[1]

BY ROBERT H. GODDARD.

WITH currents of from 0.5 to 9 amperes, there is a tendency, with contacts which show a unilateral effect, for the larger current to flow from substance to metal, even when rectification for smaller currents is in the opposite direction; together with evidences of oxidation. A large number of experiments with various substances and conditions of contact suggested the necessity of performing experiments in high vacuo, with substances cleaned in vacuo.

An apparatus in which this could be accomplished consisted, briefly, of a long glass tube with two side tubes opposite each other. The substances and metals tested were fastened to the ends of metal cylinders of a proper size to slide in these side tubes, which cylinders were, in turn, connected to platinum wires sealed in the ends of the tubes by flexible spirals of closely wound copper wire. The metal cylinder, or holder, could be moved, and rotated, from outside the tube by a magnet, acting on an iron rod in the cylinder—off center. These substances and metals could be filed, or the ends broken off, by a round file which could slide inside the large glass tube, but was prevented from breaking through by springs—one on the end of the file and another at the end of the glass tube. By this means it was found possible to break off a clean section of a soft metal, like aluminium, 1 mm. square. The vacuum was the highest that could be obtained with a Gaede pump, without freezing mixtures.

Experiments were thus performed, with as nearly chemically clean surfaces as possible, with magnesium, aluminium, iron, and lead against tellurium, silicon, galena, and mineral graphite. The effects were found to be practically independent of the particular metal used. With tellurium—melted in vacuo, to obtain a clean surface—the rectification, for currents of from 0.03 or less, to 0.1 ampere, was reduced to 2 or 3 per cent. of the current flowing through the contact, becoming for some contacts actually zero, or being slightly reversed. Ordinary fused silicon gave rectification in vacuo, but small crystals of as pure silicon as could be obtained showed rectification only of the order obtained with tellurium. Galena and carbon showed lack of uniform behavior in the air and vacuo, although the former gave noticeable unilateral effects in vacuo.

Admission of dry hydrogen, nitrogen, and carbon dioxide gave results similar to the above, but in dry oxygen the substances which had lost rectifying properties behaved as in air.

That a surface film of oxygen is necessary for rectification with metals against pure elements is a conclusion at considerable variance with some explanations

[1] Abstract of a paper presented at the Washington meeting of the Physical Society, Dec. 27–30, 1911.

that have previously been suggested, and attempts were made to check it in various ways. Tellurium was melted on a piece of sheet aluminium, and a good contact formed. The tellurium was then touched with a cold aluminium wire, and the whole allowed to cool. The result was a rectifier, the active contact being that between the tellurium and the wire. Secondly, melted tellurium, with 10 volts applied in the direction of greater resistance gave 0.5 ampere, but less than 0.0002 ampere when the E.M.F. was changed to 1.0 volt. When the 1.0 volt was applied in the reverse direction, the current increased to 0.008 ampere. With 10 volts in the other direction, no such high resistance state with 1.0 volt was observed.

Metal filings in air show difference in conductivity in the two directions, and deviation from Ohm's law—the ratio of current to E.M.F. increasing. An apparatus similar to that just described was made in which filings could be produced by the inertia of a file, shaken back and forth in the tube; there being, at the same time, practically no strain on the glass. Filings of the three metals experimented with, copper, aluminium, and magnesium, showed no deviation from Ohm's law, in vacuo or hydrogen, and no apparent asymmetry, but behaved as in air when oxygen was present. The experiments were most satisfactory in the case of copper.

As a still further check, galena, which when powdered in air shows asymmetry and deviation from Ohm's law, was filed to a powder in vacuo, and the two effects were still found to be present.

From the above experiment it is concluded that rectification is of two kinds; due to a surface film of oxygen or oxide in the case of pure elements and metals, and to a film of some other sort in the case of impure elements or compounds; which may be called surface, and body, rectification, respectively. Experiments, in which it is hoped to obtain oscillograms of rectifiers at high frequencies, to test this point further, are now in progress.

CLARK UNIVERSITY,
December 5, 1911.

A SIMPLE SLIT FOR THE SPECTROSCOPE.[1]

BY J. P. NAYLOR.

SOME time since while arranging a spectrograph for student use the writer devised a slit which is believed to be new in construction and of sufficient merit to be of interest to readers of the REVIEW who are engaged in spectroscopic work. Besides having several advantages over a slit made in the usual way, this slit is so simple in construction that it can be made by anyone having ordinary mechanical skill and using only very simple tools. And yet the slit, by a little care in the making, is suitable for the most exacting work.

The base on which the slit is arranged consists of a piece of one eighth inch sheet brass, one side of which is made flat by grinding it on a piece of emery paper glued to a flat block of wood. A hole of the necessary size to limit the length of the slit is drilled in the center of the base and over this opening the plates are ar-

[1] Abstract of a paper presented at the Washington meeting of the Physical Society, Dec. 27–30, 1911.

ranged that form the jaws of the slit. These plates are made of No. 16 sheet brass or, better still, German silver. The plates are carefully flattened by grinding on the emery paper.

One of the plates, *A* of the figure, is fastened to the base by two flat head screws which pass through holes in the plates which are slightly slotted to permit of a small lateral adjustment of the plate.

The other plate, *B*, is movable so as to regulate the width of the slit. This plate has its two edges inclined to each other in the ratio of 10 to 1. The back edge of this piece can slide endwise under the flat heads of two screws, 2 and 3, so placed as to bring the front edge about parallel with the edge of the fixed plate. A similar screw is fastened in the center of the plate and the head of the screw is raised so as to permit a piece of No. 26 spring brass wire to pass under it. This wire has its ends bent under two screws, 7 and 8, near the ends of the base and forms a spring that holds the plate down to the base and back against the body of the screws, 2 and 3. The opposed edges of the plates are beveled back slightly from the front and are, of course, carefully straightened by filing and finally by grinding on plate glass, first, with fine emery and last with powdered pumice.

To make the edges of the slit perfectly parallel screws 5 and 6 are slightly loosened and the edge of *B* is gently forced down against *A* when, if the screws are again tightened, the edges will remain parallel provided the back edge of plate *B* has been ground straight. A millimeter scale and an index on *B* is provided by which the width of the slit can be readily determined. This is a convenience usually only found on expensively constructed slits provided with a micrometer screw and divided head.

The facility with which the slit can be cleaned is an advantage which will certainly appeal to workers with the spectroscope. By simply slipping off the wire spring plate *B* can be removed; the edges can then be cleaned and the plate returned accurately to its former position. If the edges, by any accident or use, should become damaged the plates can be easily removed, the edges reground, and the plates put back in a very short time.

It will be observed that, by using two movable plates arranged as above, the construction readily permits of the making of a bilateral slit. In this case, the second plate should be backed up with a brass plate provided with screws and slotted holes so as to permit of adjustment for parallelism of the jaws. If by means of a pin on one plate and a projection on the other the plates are made to move together the slit will open symmetrically.

MINSHALL LABORATORY,
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THE DISTRIBUTION OF THE ACTIVE DEPOSIT OF RADIUM IN AN ELECTRIC FIELD.[1]

BY E. M. WELLISCH AND H. L. BRONSON.

IN this paper an account is given of experiments which have been made to determine the activity that is transmitted to the electrodes in an electric field. This activity has throughout been obtained for each electrode as a percentage of

[1] Abstract of a paper presented at the Washington meeting of the Physical Society, Dec. 27–30, 1911.

the total activity. It has been found that the curve connecting the percentage of the total activity transmitted to the cathode with different electric fields has all the characteristics of the curves for ionization by α-particles. The anomalous results obtained by many previous experimenters are almost certainly due to the fact that the electric fields employed were far from saturating. It has been found that all the so-called anode activity is due to the diffusion of uncharged carriers.

It has been shown that the curves for α-ray ionization do not in general have asymptotes such as the ordinary saturation curves for X-ray ionization, in fact, the more nearly the curve approaches to parallelism with the axis of electric field the farther is the ionization from saturation. The shape of the ionization curves has been explained as being due to the formation by the α-particle of "neutrons" which are afterwards separated into ions by the collision with ions already established in the columns. The results obtained by several workers in the case of different inclinations of the α-ray columns are readily explainable on the present hypothesis.

THE THOMSON EFFECTS IN AND THE THERMAL CONDUCTIVITIES OF TUNGSTEN, TANTALUM AND CARBON AT GLOWING TEMPERATURES.[1]

BY A. G. WORTHING.

IN the investigation of the losses in incandescent lamps due[2] to the cooling effects of the supports and the leading-in wires, a variation was noticed which was ascribed to the Thomson effect. The suggestion was made by Dr. Hyde at that time that the same method might be used to study the Thomson effect. This has been done. The method of observation is the same as that of the investigation noted above, though greatly refined as to accuracy. The observations consist essentially in the determination by an optical pyrometric method, in the case of an incandescent lamp, of the time rate of energy radiation per unit of length for elements of filament in the neighborhood of a support or leading-in wire as a function of their distances from the support or leading-in wire; (*a*) when the current through the element of filament is flowing toward the cooling support; (*b*) when in the opposite direction. The difference between these two quantities is in part due to the Thomson effect. From the platted data as a basis, as soon as the temperature of an element is known in terms of the time rate of energy radiation per unit of length, the Thomson effect may be determined by means of the following equation, when elements of filaments at the same temperature are chosen:

$$\sigma = \frac{W_a - W_b}{I \frac{d\theta}{dE}\left[\left(\frac{dE}{dl}\right)_a + \left(\frac{dE}{dl}\right)_b\right]},$$

where σ represents the Thomson E.M.F. per unit of temperature difference,

[1] Abstract of a paper presented at the Washington meeting of the Physical Society, December 27–30, 1911.

[2] Hyde, Cady and Worthing, Trans. Ill. Eng. Soc., 6, 1911, p. 238; Hyde, Cady and Worthing, Ill. Eng., Lond., 4, 1911, p. 389.

W the time rate of transformation of electrical energy into heat per unit length, I the current, θ the temperature, E the time rate of energy radiation per unit of length, l the distance of the element in question from the support or leading-in wire, and a and b, as subscripts, the two conditions as to direction of current noted above.

The thermal conductivities of the substances may be obtained from the same data as given above. Here the equaton is

$$k = \frac{\int_l^{l_m} (W_a - E_a)dl}{S\,\frac{d\theta}{dE}\left(\frac{dE}{dl}\right)_a} = \frac{\int_l^{l_m} (W_b - E_b)dl}{S\,\frac{d\theta}{dE}\left(\frac{dE}{dl}\right)_b},$$

where in addition to the above symbols k represents the thermal conductivity, l_m the distance to a point where the rate of heat conduction along the filament is negligibly small, and S the cross section of the filament. Thus far the thermal conductivities have been expressed only in terms of their ratios to the corresponding electrical conductivites.

The lamps used were kindly calibrated by Professor Mendenhall and Mr. Forsythe as to temperature in the neighborhood of the temperatures at which they have their ordinary efficiencies. Other temperatures were obtained by using certain results not yet published which are due to Dr. E. P. Hyde. The uncertainties arising from temperature measurements in this work will not change the order of magnitude of the results obtained. Some results are included in the following table.

Absolute Temperature.	Tungsten.	Tantalum.	Carbon.
Coefficients of Thomson effect in $\frac{\text{microvolts}}{\text{deg.}}$.			
1,600° K	16	—5	—20
1,800	22	—6	—20
2,000	29	—8	—20
2,100	33	—9	—20
$10^{-10} \times \frac{\text{thermal conductivity}}{\text{electrical conductivity}}$ in C.G.S. electromagnetic units.			
1,600° K	56	27	280
1,800	73	33	270
2,000	94	46	260
2,100	103	56	240

When these results together with some thermoelectric results on tungsten and tantalum by Coblentz[1] are handled thermodynamically, it is found that a tungsten-tantalum thermocouple at high temperatures is about four times as sensitive as a platinum-platinum rhodium thermocouple.

It is evident that the Peltier E.M.F. between two metals may be measured

[1] Bul. Bur. Stds., 6, 1909, p. 107.

by a method similar to the foregoing. All that is essential is to determine the change in the rate of heat conduction along a filament composed of the two metals welded together, in passing from one side of the junction to the other. Such measurements are contemplated.

THE DIELECTRIC CONSTANT, RESISTIVITY AND ELECTROSTATIC ABSORPTION OF DIFFERENT CRYSTALS.[1]

BY H. L. CURTIS.

IN 1881, Rowland and Nichols measured the electrostatic absorption in plates of mica, quartz, selenite and calcite. They found no absorption in calcite, but their results do not show how the absorption of the other substances compares with mica. It seemed desirable, therefore, to measure the absorption of a larger number of crystals, and to compare their absorption with that of mica which has been extensively studied. Also it was thought best to measure the dielectric constant and resistivity of the crystals, thus giving important electrical data concerning them.

Thin plates of the crystals were used as the dielectric in a plate condenser having a guard ring. The same arrangement was used for determining the resistivity.

Two method of measuring the capacity of the condenser were used; one by direct current and the other by high frequency alternating current. In the direct current method, the condenser is charged 0.5 second, then disconnected from the battery, and about five hundredths of a second later is connected to a tenth microfarad air condenser for a few thousandths of a second. As the capacity of the air condenser is much larger than the condenser which is being measured, nearly all of the charge is transferred to the air condenser. The charging and discharing is accomplished by a mechanism driven by a motor, the gearing being so arranged that the operation of charging and discharging is repeated every two seconds. The air condenser is discharged through a ballistic galvanometer when a sufficient quantity has collected to make a reasonable deflection. A variable air condenser is then substituted for the plate condenser, and its capacity varied until, with the same number of discharges, the deflection of the galvanometer is the same as with the plate condenser. From the measured capacity and the capacity computed with air as a dielectric, the dielectric constant is determined.

In the apparatus just described, a contact is arranged so that a second discharge of the condenser takes place either 0.5 or 1 second after the original discharge. This discharge can be collected upon the air condenser instead of the first discharge. From the measurement of this discharge, the absorption under the given conditions is computed.

In cases where the resistivity is so low that the method just outlined does not give satisfactory results, an approximate measurement of the dielectric

[1] Abstract of a paper presented at the Washington meeting of the Physical Society, December 27–30, 1911.

constant can be obtained from high frequency measurements of the capacity. The plate condenser without a guard ring is placed in parallel with a variable air condenser in a resonating circuit with a known inductance, and the frequency determined by means of a wave meter. The crystal condenser is then removed and the air condenser varied until the resonating circuit has the same frequency as at first.

The resistance is obtained by means of leakage methods such as are commonly used for high resistances. As a guard ring is employed, the measured leakage takes place through the crystal and not over its surface.

Results have already been obtained upon a number of crystals and others will be investigated. The following table gives some of the values which have already been determined. The maximum and minimum value of the index of refraction, determined by the angle of total reflection from the surface, has been given for most of the crystals.

The results show that fluorite has no measurable absorption; *i. e.*, less than 0.1 per cent. The calcite which was used showed some absorption, but the plate was not perfect, having several cleavage cracks across its face. Also the dielectric constant of rutile is much smaller than that found by other observers.

Crystal.	Capacity in Micro-microfarads.	Dielectric Constant.	Index of Refraction for Sodium Lines.		Absorption.		Resistivity in Megameg-ohm-cm.
			Max.	Min.	0.5 Second Discharge.	1 Second Discharge.	
Mica.....	347.	6.8	1.560	1.598	0.26%	0.30%	80,000.
Fluorite.....	5.3	6.9	1.434		0.	0.	3,000.
Calcite.....	15.5	8.3	1.658	1.486	2.7	3.7	1,600.
Quartz.....	70.	4.9	1.553	1.544	11.	13.	69.
Selenite.....	13.3	4.8	1.531	1.522	4.3	6.5	1,400.
Green beryl ..	14.4	5.6	1.586	1.580	3.9	4.4	200.
White beryl ⊥.	27.6	7.4	1.578	1.571	22.	25.	17.
White beryl ‖..	25.9	9.3	1.580		8.5	9.2	160.
Cryolite.....	15.2	6.2	1.335		8.6	10.	200.
Apatite.....	16.9	7.8	1.654		12.	13.	1,200.
Barite.....	20.6	9.0	1.647	1.635	21.	24.	6.
Spodumene...	35.3	17.6	1.713	1.686	25.	27.	6.
Monocline....	14.8	5.8	1.519		27.	32.	23.
Feldspar.....	17.7	5.7	....		26.	29.	24.
Celestite.....	11.7	6.3	1.626	1.619	31.	37.	380.
Talc.....	13.0	5.8	....		32.	36.	50.
Rutile.....	240.	31.	2.903	2.616			0.09

THE INFLUENCE OF TEMPERATURE ON THE PHENOMENA OF PHOSPHORESCENCE IN ZINC SULPHIDE.[1]

BY HERBERT E. IVES AND M. LUCKIESH.

THREE phenomena have been the subject of study, namely, the law of decay, the flashing up of brightness under infra-red, and the flashing up with heating. Temperatures were varied between − 80° C. and + 200° C. The decay curves obtained, if plotted in terms of $1/\sqrt{I}$ against time, vary from strongly concave to the time axis at low temperatures to strongly convex at high temperatures. The formula

$$I^{\alpha} = a + bt$$

fits all the curves obtained, where α varies from − 1.6 to − .2 over the range of temperatures employed. This formula holds from two or three seconds after the end of excitation to fifteen minutes, the longest time so far investigated. The initial intensity, or fluorescence, is not however given by the formula, which gives too low a value. The change from fluorescence to phosphorescence obeying the above relationship appears to be almost instantaneous.

The flashing up of brightness under infra-red, which the writers have found to be greater the longer the interval since excitation, is now found to be greater at low temperatures and less at high.

Heating always causes a flash of light. In the early stages of decay where infra-red causes an immediate drop in brightness, heat and infra-red act oppositely. An experiment with separate and then simultaneous application of heat and infra-red shows the two to sum algebraically. This is not in harmony with the theory that infra-red is equivalent to high temperature.

The facts here presented are not compatible with the theory that the law of decay is represented by the expression

$$1/\sqrt{I} = a + bt,$$

or by a summation of such terms. Such a summation can account only for curves concave to the time axis (on a $1/\sqrt{I}$ against time plot), not for curves convex to it.

A possible explanation of the experimental facts is that the sample of zinc sulphide is a mixture of components all decaying according to some simple law. Curves of the character found are given by the summation of several terms of the form Ae^{-kt}, which is the law of mass action with one substance changing. k varies with temperature. Since different samples of zinc sulphide decay at different rates at the same temperature it is possible that the variation of the k's is with respect to temperature zeros which are functions of the composition of the components.

According to this view the later part of a decay curve is due to components at a "low" temperature; the flashing-up phenomenon, a low temperature effect, therefore, becomes more marked as decay goes on.

[1] Abstract of a paper presented at the Washington meeting of the Physical Society, December 27–30, 1911.

Spark Spectra in the Schumann Region.[1]

By Theodore Lyman.

THE spark spectra of metals in the region of short wave-lengths have been investigated by Schumann and by Handke.[2] In both researches the spark was placed in air outside the vacuum spectroscope. Handke, who worked with aluminum, copper, gold, silver, zinc, magnesium and mercury in this way, made measurements of wave-length as far as the neighborhood of λ 1600.

In the work described in the present paper an improvement was introduced by placing the spark in a stream of pure, dry hydrogen at atmospheric pressure. The mantle of absorbing gases around the spark was thus very much reduced, with the result that the spark spectra of aluminum, magnesium, calcium and strontium-amalgam yielded lines up to the limit set by the transparency of fluorite, λ 1250. The spectrum of calcium proved of particular interest since it showed some new series relations. The work on the spectra of the alkali earths is being continued.

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A New Form of Mechanical Vacuum Pump.[1]

By John Johnston.

THIS pump—known as the May-Nelson Rotary Pump—consists essentially of two plates of metal, one fixed, the other movable; no valve, packing or fluid is required, though a little lubrication is desirable for mechanical reasons. A somewhat defective pump of this type has given a vacuum of 0.01 mm. when connected to the gage by rubber tubing. It has exhausted 6 liters to 0.1 mm. in 2 minutes. After pouring in 1 c.c. of water, the pressure was reduced to 0.5 mm. in 4 minutes, without drawing any air through the pump. The exact mode of working is difficult to show without a diagram, but the principle may perhaps be made sufficiently clear by what follows:

In each of the metal plates are cut a series of concentric grooves, which are so situated that the projections on each plate will lie in the grooves on the other plate. Through the center of the fixed plate passes a shaft, on which is an eccentric carrying the movable plate. Rotation of the shaft thus produces an eccentric motion—but no rotation—of the movable plate; as a result, there is, between each adjacent pair of curved surfaces, a rolling contact which sweeps out the air ahead of it. The pump above mentioned has an inner diameter of 13 cm., a depth of 4 cm., and consists of three rings which pump in series; the speed of rotation of the shaft was 800 r. per minute; ¼ H.P. suffices to operate it.

The effectiveness of this form of pump appears to be due to two things: (1)

[1] Abstract of a paper presented at the Washington meeting of the Physical Society, December 27–30, 1911.

[2] Inaug. diss. u. Berlin, 1909.

A very slight travel of the metal parts produces a relatively rapid motion of the crescent-shaped spaces between the rings on the two plates; (2) the contacts, since they occur between curved surfaces of nearly the same radius, are relatively long; hence the viscosity of the air is sufficient to prevent appreciable leakage backwards.

This pump may also be operated as a blower, or, with slight modifications, can be used as a water motor; indeed, since the rings may be made independent of one another, one can drive the pump by supplying water to one ring and obtain vacuum or blast from the other rings.

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THE PHOTOELECTRIC EFFECT OF PHOSPHORESCENT MATERIAL.[1]

BY CHESTER A. BUTMAN.

PHOSPHORESCENT CaS prepared according to the method of Lenard and Klatt and also Sidot blende were examined for the photoelectric effect in a high vacuum.

1. It was found that the photoelectric fatigue and recovery was an inherent property of the material. The fatigue is due solely to the incident light.

2. The velocities of the electrons ejected from these substances is dependent on the photoelectric state of the material. The greatest voltage necessary to keep the electrons from coming off from the CaS was -0.35 volt. In a fatigued state it was sometimes necessary to apply $+4$ volts in order to get any photoelectric effect.

3. The phosphorescent intensity was found to be approximately independent of the state of the photoelectric fatigue.

4. Infra-red light gave no photoelectric effect.

5. The components of the phosphorescent CaS were examined for the photoelectric effect, and none were found to be active with light of longer wave-length than 3,200 Å.ng., with the exception of sulphur. It was found that light passed through glass which gave no effect with brass, lead, or platinum, did give a photoelectric effect with flowers of sulphur and brimstone.

6. The following theory is proposed: Phosphorescent light is the light emitted at the time when a charged atom collides with the material that emits the light which is characteristic of that phosphorescent compound. The charge on the atom comes from the electrons sent off from the sulphur. This theory only applies to compounds where there is no phase relation between the exciting and excited light. It is possible to prepare a solid fluorescent (short duration) compound without sulphur. However, here we probably have to do with a *selective emission* due to resonance. This last would be proven to be the case if a phase relation was shown between the exciting and excited light, as is the case with liquids.

[1] Abstract of a paper presented at the Washington meeting of the Physical Society, December 27–30, 1911.

7. The experiments of Ives showing a difference in the action of heat and infra-red would be interpreted as follows: The heat causes more collisions to take place making a flashing up. The infra-red has little effect on the atomic movements, but does dampen the period of the oscillating electrons in the emitting material, hence the extinguishing action.

Note on the Ascensional Rate of the Free Balloons Used for Meteorological Purposes.[1]

By Wm. R. Blair.

THE same type of balloon was used for a number of ascensions. From these ascensions, 20, all of those reaching an altitude of 18 or more kilometers above sea level, were selected, and the ascensional rates of the balloons considered in connection with other data obtained in the ascensions.

It is found that the effect of temperature on the relative densities of the gases outside and inside of the balloon, and consequently, on the ascensional rate of the balloon, needs more consideration than has been given it in the note by A. Mallock, F.R.S., on the "Ascent of Meteorological Balloons and the Temperature of the Upper Air," Proc. Roy. Soc., A80, p. 530. It is also found that the variations in the ascensional rate of these balloons is so great that computations based on the assumption of uniform ascensional rate are much in error, especially at the higher altitudes.

The Vertical Temperature Gradient of the Atmosphere.[1]

By Wm. R. Blair.

THE data considered are those collected by the Blue Hill Meteorological Observatory at St. Louis, Mo., and at Pittsfield, Mass.—59 observations, and those collected by the Mount Weather Observatory at Fort Omaha, Indianapolis, and Huron—79 observations.

These data, together with considerations of convection, nucleation, and absorption, radiation and conduction of heat by the atmosphere seem to point to the conclusion that, while the temperatures themselves are determined mainly by absorption and radiation of heat supplied by the radiators on either side of the atmosphere, the vertical temperature gradient as far up as it has been observed owes not only its local peculiarities but its general character to convection and closely related phenomena. There is evidence that at no great distance above the explored region, the character of the vertical temperature gradient is determined chiefly by gravitational arrangement of the atmospheric constituents.

[1] Abstract of a paper presented at the Washington meeting of the Physical Society, December 27–30, 1911.

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